MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Ernest Agar Beet, B.Sc., Epworth College, Rhyl, Flintshire ;
John Morton Billing, Public Works Dept., Johore Bahru, Federated Malay States ;
Francis Gilbert Brown, 13 Peak Hill Gardens, Sydenham, London, S.E. 26 ;
Commander Evan P. Cameron, R.D., R.N.R., 129 Kenilworth Court, Putney, London, S.W. ;
Jacques François Jean Gérard Cox, D.Sc., Assistant Professor, Brussels University, 139 Avenue Buyl, Brussels, Belgium ;
Robert Tait Duthie, Alexandra Terrace, Fraserburgh, Aberdeenshire ;
Sir Patrick Fagan, K.C.I.E., C.S.I., 73 Canfield Gardens, London, N.W. 6 ;
Emrys Howells, M.Sc., The Technical College, Cardiff, and 4 Llandough Street, Cardiff, S. Wales ;
Anton Gotfred Hoyer, Bwana Mkuba, N. Rhodesia ;
Diwan Ramchandra Kapur, M.A.,S.B., F.R.S.A., Diwan Balmokund Kapur Lane, Benares City, U.P., India ;
James Duncan McNeile, M.A., Wellington College, Berks ;
George Patrick Meredith, M.Sc., 13 Elmwood, Welwyn Garden City, Herts ;
Louis Rosenhead, Ph.D., St. John's College, Cambridge ;
William Valentine Shearer, B.Sc., Whitworth Scholar, A.M.I.C.E., M.I.M.E., City and Guilds (Engineering) College, Imperial College of Science and Technology, London, S.W. 7 ;

Jacques Simard, B.A., M.B.A., 1566 St. Denis Street, Montreal, Canada ;
René Simard, S.B., S.M., 1566 St. Denis Street, Montreal, Canada ;
Charles H. Smiley, M.A., Ph.D., 18 Kidbrook Park Road, London, S.E. 3 ;
Robert Gidner Spencer, D.L.C., The School House, Gembling, E. Yorks ;
Cyril Longford Twentyman, 21 Bathhurst Road, Grassendale, Liverpool ;
James Ernest Grant Yalden, 120 Woodridge Place, Leonia, N.J., U.S.A. ; and
Jacob Werner, Imperial Hotel, Piet Retief, Transvaal, S. Africa.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Cyril Jackson, Union Observatory, Johannesburg, S. Africa (proposed by H. E. Wood) ;
J. E. Kerrich, M.Sc., University of the Witwatersrand, Johannesburg, S. Africa (proposed by R. T. A. Innes) ;
Antonie Pannekoek, Professor of Astronomy, University of Amsterdam (proposed by W. de Sitter) ;
Dhirendra Nath Saha, Extra-Assistant Superintendent, Survey of India, Karanpur, Dehra Dun, U.P., India (proposed by S. G. Burrard) ;
Saunp Chand Mahanlal Shah, B.A., 7 Green Lane, London, N.W. 4 (proposed by L. N. G. Filon) ; and
Gustav Stracke, Recheninstitut, Berlin-Dahlem, Germany (proposed by F. W. Dyson).

Sixty-eight presents were announced as having been received since the last meeting, including, amongst others :—

Astrographic Charts : Bordeaux Observatory, 15 charts (zones $+12°$, $+14°$, $+16°$) ; Paris Observatory, 17 charts (zones $+18°$, $+20°$, $+22°$).
Tycho Brahe, Opera Omnia, Tomus xv. (Index to vols. 1–14), presented by Admiral Dreyer.
Mount Stromlo, Commonwealth Solar Observatory, Memoirs, No. 1 (2 copies).
Nice Observatory, Catalogue de 1020 étoiles " Intermédiares."
M. Proctor, Romance of the Planets.

The President announced that the Gold Medal of the Society had been awarded to Dr. J. S. Plaskett, for his valuable observations of

stellar radial velocities and the important conclusions derived from them.

The President further announced that Dr. Plaskett had been invited to deliver the George Darwin Lecture for 1930.

The President read the following resolutions of the Council, which were approved by the meeting :—

The Council desire to express their sorrow at the death of Major P. A. MacMahon, who was a Fellow of the Society for thirty-four years, during eighteen of which he served on the Council, being Vice-President for seven years and President from 1917 to 1919. They request the President to convey their sympathy to his widow, and to assure her of the high esteem in which Major MacMahon was held by the Society.

The Council desire to express their grief at the tragic death of Major P. H. Hepburn, who was a member of Council from 1922 to 1928, being Treasurer from 1927 to 1928. His legal knowledge was frequently of service to the Council. His chief astronomical study was the planet Saturn, and for many years he was director of the Saturn section of the British Astronomical Association. The Council request the President to convey to his widow and family their sincere sympathy in their sad bereavement.

The Motions and Distribution of Interstellar Matter.

By J. S. Plaskett and J. A. Pearce. (Plate 9.)

Introduction.

Some twenty-five years ago Hartmann * first observed, in the spectroscopic binary δ Orionis, that the H and K lines of calcium did not share in the velocity oscillations of the hydrogen and helium lines but remained relatively fixed in position, hence the name " stationary " calcium lines, which has served for their designation almost to the present time. For nearly twenty years, although many more examples of spectroscopic binaries with " stationary " calcium lines were found, it was generally believed that they were associated only with binaries, and it was not recognized that they were also present in other high temperature stars. The observational data in regard to these " stationary " lines was summarized by R. K. Young,† who reached the conclusion that they were due to the absorption of a surrounding cloud of ionized calcium, which was sometimes set into revolution in the same period, but with smaller amplitude, by the binary motion of the stars through the cloud.

* *Ap. J.*, **19**, 268, 1904.

† *Pub. D.A.O.*, **1**, 219, 1920.

A new phase of the problem developed in a research of one of the authors, J. S. Plaskett,* on the O-type stars. It was conclusively shown that the "stationary" calcium lines, differentiated from the other star lines by their extreme narrowness and sharpness, were present not only in spectroscopic binaries but in all O-type stars, even including the Wolf-Rayet stars in which no other absorption lines were present. The velocity resulting from the calcium lines generally differed, and frequently markedly, from the velocity given by the other lines in the spectrum. These velocity differences were so great, up to 60 km. per second, in comparison with the probable errors of the determinations, frequently only about a kilometre per second, as to leave no possible doubt of the relative motion of the star and the absorbing material which produced the stationary lines. Further, when it appeared that the calcium velocities generally differed little from the reflex of the solar motion, the conclusion was hardly escapable that these high temperature stars were in motion, frequently in rapid motion, relative to or through clouds of ionized calcium which were, comparatively speaking, at rest with respect to the local cluster. As at that time the "stationary" lines had not been identified with any stars of type later than B3, it was supposed this interstellar material was ionized and rendered absorbing at H and K by the excitation of the stars of highest temperature.

A great stimulus to the problem of this interstellar matter was given by Eddington † in the 1926 Bakerian lecture of the Royal Society. He discussed theoretically the physical conditions present in diffuse matter in interstellar space and showed that it was probably uniformly distributed, except where condensations produced the diffuse nebulæ, and had a density of 10^{-24} gm./cm.^{3} and a temperature, defined by the molecular speed, of 10,000 or 12,000 degrees absolute.‡ At this temperature most of the calcium present would be doubly ionized, the sodium singly ionized, calcium and sodium being the only elements likely to be detected, but neither of these ionized atoms give any lines in the observable region. There would be, however, one part in 3000 of Ca^+, but only one part in 2,000,000 of *Na* and one in 2,000,000,000 of *Ca*. It was calculated that this quantity of Ca^+ would be sufficient to produce appreciable absorption of H and K in a depth of 100 parsecs, but the appearance of interstellar D lines was unexplained. It was pointed out that it was unnecessary to assume excitation from the high temperature stars, which would tend to decrease the amount of Ca^+ present by changing it into Ca^{++}.

On Eddington's hypothesis, therefore, the presence of this diffuse material in interstellar space should have two consequences as regards the stationary, or, as we prefer to call them, the "interstellar" Ca^+ lines H and K. First, these lines should be present in the spectra of

* *Pub. D.A.O.*, **2**, 335, 1924; *M.N.*, **84**, 80, 1924.

† *Proc. Roy. Soc.*, **111**, 424, 1926.

‡ Gerasimovic and Struve in *Ap. J.*, **69**, 7, 1929, have calculated that the density of the interstellar material is about 10^{-26} gm./cm.^{3}, one-hundredth of Eddington's value, and the temperature about 15,000°.

all classes of stars, and secondly, they should increase in absorption or strength with the increase in distance of the star. It would, therefore, be of considerable value to test observationally these two consequences of Eddington's hypothesis.

So far as the first consequence, the presence of these lines in all spectral types, is concerned, it will be seen later that, while "interstellar" lines appear in some B3 and B5 stars, they have not yet been detected in any later types. Detection in B8, B9, or A-type stars will be difficult owing to the presence of Ca^+ absorption in the stellar atmosphere itself, while in F and later types it will be impossible owing to the width and strength of the stellar Ca^+ lines H and K. Stellar H and K are plainly visible in some B3 and B5 stars, are strong and well marked in B8 and B9, with rapidly increasing strength and width in A stars. The detection and discrimination of "interstellar" from stellar calcium in stars later than B5 would require, first of all, relatively strong "interstellar" lines, this entailing on Eddington's hypothesis great distance and hence faint objects, difficult to observe spectroscopically. Secondly, in order to discriminate between "interstellar" and stellar calcium, the latter must be displaced by the Doppler effect by an amount equivalent to a velocity of about 100 km. per second before the two lines can be separated with the low dispersion necessary for faint stars. The number of B8, B9, or early A stars likely to have a peculiar velocity of 100 km. or greater, or of binaries of the same classes with a range of 200 km. or over, caught at the time of maximum displacement, is relatively small, and the chance of certainly finding "interstellar" calcium in later types than B5 correspondingly remote.

As regards the second necessary consequence of Eddington's hypothesis of uniform distribution of the interstellar material, that the absorption of this material and the strength of the "interstellar" calcium lines must increase with the distance of the stars, the position is a little more hopeful. Struve has already recognized this fact and presented two able papers in the *Astrophysical Journal* on the relation between the intensity of the "interstellar" calcium lines and the faintness and hence, presumably, the distance of the stars. In the first of these papers* he correlated the intensities of the "interstellar" K line, H on account of blending with $H\epsilon$ being unsuitable for intensity estimates, with the apparent magnitudes of B- and O-type stars observed at Yerkes, Mt. Hamilton, Mt. Wilson, and Victoria. From these estimates he showed that there was an increase in the strength of the "interstellar" K line as the stars became fainter until about magnitude 7, while for stars fainter than the 7th magnitude there was a decrease in intensity, this decrease being in opposition to Eddington's hypothesis. These results, however, are based on relatively few faint stars and, moreover, the material was heterogeneous and made up of spectra from four different observatories.

In Struve's second paper,† however, the objection of insufficiency of data for the fainter stars is removed by using the objective prism spectrograms of Harvard Observatory. The intensities of "inter-

* *Ap. J.*, **65**, 163, 1927. † *Ibid.*, **67**, 353, 1928.

stellar" K were estimated in 1718 stars of spectral types O to B3, ranging in magnitude down to about 10·5. The average intensities of K in varying magnitude groups of these 1718 stars increased linearly from intensity 1 for first magnitude stars to intensity 4·4 at magnitude 10·5, the plot showing almost a straight line over the whole range. The decrease at magnitude 7, given in the first paper, had entirely disappeared, and these results show a definite increase in strength of "interstellar" K with increase in apparent magnitude, thus generally confirming Eddington's hypothesis of uniform distribution of the interstellar material. There are, however, in these results some uncertain factors, and caution in their complete acceptance is desirable. Thus the material is decidedly not homogeneous, as the spectra were obtained by objective prism instruments with dispersions ranging from 36 to 320A per millimetre. The known dependence of the character of the lines of objective prism spectra on the conditions of focus, etc., and on the quality of the seeing, may easily produce great differences in spectra of the same star made with the same dispersion, to say nothing of the enormous differences inherent in spectra of 36A and 320A per millimetre dispersions. Such differences would seem to make the intercomparison of the intensities of such a narrow absorption line as K with different dispersions extremely uncertain. Indeed, Struve himself says that in the low dispersion spectra the two lines of *Si* III at 3924 and K at 3934 "are almost blended, and this caused considerable uncertainty in the estimates."

It would be desirable, therefore, that an independent determination of the manner in which the interstellar material is distributed, whether uniform or not, should be undertaken. It would also be desirable if this determination did not primarily or wholly depend upon estimates of the intensity of the "interstellar" K line. Such estimates are difficult and uncertain on account of the general weakness of the continuous spectrum in that region. It is proposed in this paper, therefore, to determine the distribution of the interstellar material by an entirely different method, a method based on a determination of the relative distances of the star and of the centre of absorption in the cloud. This method analyses the stellar and interstellar velocities with reference to a differential rotation of the galactic system.

The theory of a rotation of the galaxy was introduced by Lindblad * in a series of papers, and was developed by Oort,† who investigated its consequences on the proper motions and radial velocities of the stars, and showed the probability of such a differential rotation. Further evidence was given by one of the writers, J. S. Plaskett,‡ who showed that the differential radial motions that would be produced by galactic rotation were markedly present in the radial velocities of about 550 O and B stars. The rotational term $\bar{r}$A, determined from the radial motions of a group of stars, consists of a constant part A, which has a value of about ·017 km. per second per parsec, and the variable part $\bar{r}$,

* *Medd. Upsala*, Nos. 3, 4, 6, 13, 1925, 1926; *M.N.*, **87**, 553, 1927.
† *B.A.N.*, Nos. 120, 132, 133, 1927.
‡ *M.N.*, **88**, 395, 1928.

which is the mean distance in parsecs of the group of stars from which $\bar{r}$A is determined. It is the possibility of determining the mean distance of a group of stars, or of the centre of gravity of the absorption of the calcium clouds, which makes it possible to test the distribution of these interstellar clouds by the galactic rotation.* As a necessary preliminary, however, the rotation of the galaxy will be further tested by the radial velocities of the interstellar clouds, in order to confirm its applicability in obtaining mean distances.

I. *The Motions of the Interstellar Clouds.*

Observational Material.

The material for this discussion has been gradually accumulating for the past five years in the determination by the authors of the radial velocities of all Bo to B5 stars of visual magnitude 7·5 or brighter, and north of declination − 11°, which had not been previously investigated. There were about 450 stars in this programme, mostly fainter than 5·5 magnitude, and, in order that the velocities should be reliably determined, an average of about five plates per star was obtained. For many stars with variable velocities more plates were secured to enable an approximate value of the velocity of the system to be obtained. It is believed that the velocities, both of the stars and of the interstellar clouds in the line of sight from the stars, are reliable. The average probable error of the determination of the radial velocity of these B-type stars is ± 1·80 km. per second, while that of the interstellar clouds is slightly less, ± 1·74 km. per second.

It will be of interest to examine in more detail the probable errors of the radial velocities of both stars and interstellar clouds. The material selected comprised 235 stars, for which both stellar and interstellar velocities were determined, which were used in one group of solutions. In the table the column headed "Undetermined" refers generally, for the stars, to spectroscopic binaries with a γ velocity adopted from the mean of about eight observations, and, for the clouds, to interstellar velocities determined from only one or two measures. None were included of large range or otherwise uncertain, and the probable errors are not likely to exceed those of the preceding column. As both the dispersions used with these stars give a flat field and beautiful definition over the entire photographic region from below λ3800 to λ5000 or beyond, and as the number of lines measured per plate averages more than eight, it is believed that the velocities of

* Gerasimovic and Struve (*Ap. J.*, **69**, 9, 1929), in the last section of their paper on "Physical Properties of a Gaseous Substratum in the Galaxy," have discussed the galactic rotation of the interstellar matter from the Ca^+ velocities of 103 stars. The *r*A of the cloud of + 5·3 km. is compared with that of the stars of + 12·0 km. and the relation of 1 to 2·3 taken as evidence of approximately uniform distribution of the interstellar matter. The $\bar{r}$A for the stars instead of being directly computed was, however, merely interpolated from Plaskett's (*M.N.*, **88**, 395, 1928) values, and this with the somewhat heterogeneous data makes the conclusion not very convincing.

both stellar and interstellar lines are reliable, and that they form a much more accurate and homogeneous system than any previously used.

Table I.

Probable Errors of Radial Velocities.

Range of Errors.	0 to 1·5.	1·5 to 2·5.	2·5 to 3·5.	3·5 and greater.	Undetermined.
Nos. of Stellar velocities.	98 (42%)	42 (18%)	18 (8%)	18 (8%)	59 (25%)
Average Probable Error.	± 0·85	± 2·07	± 3·15	± 5·03	
Nos. of Interstellar Velocities.	100 (43%)	47 (20%)	24 (10%)	14 (6%)	50 (21%)
Average Probable Error.	± 0·91	± 2·10	± 2·95	± 4.40	

Average Probable Error, 176 Stellar Velocities = ± 1·80 km. per sec.
Average Probable Error, 185 Interstellar Velocities = ± 1·74 km. per sec.

Of the 500 or more B- and O-type stars observed here, nearly 300 had pronounced Ca^+ lines. In selecting from these 300 the stars which contained " interstellar " lines, two criteria were employed—the character of the H and K lines, and the relation or the difference between the stellar and the interstellar velocity. The characters of the " interstellar " lines are so different from those which arise in the stellar atmosphere that, in the great majority of cases, there is no possibility of mistakes. The " interstellar " lines are sharp, narrow, and strong, with sharply limited edges and no appearance of wings. They have a distinctive, clear-cut appearance, quite different from any other lines present in O- or B-type stars, and, indeed, are much more clear cut than lines in late-type spectra, the nearest approach in character being in emission hydrogen lines with strong absorption reversal. In all stars of types O to B2, when Ca^+ lines are present there can be no doubt that they are due to interstellar absorption, as no stellar Ca^+ lines have been seen in any of our plates of these types.

When, however, we come to types B3 and B5 the matter is not so simple, though the stellar can usually be told from the " interstellar " line by the sharp, clear-cut appearance of the latter. The only doubtful cases are the stars of these types which have strong and sharp stellar lines of helium, etc. These are probably " *c* " stars, super-giants, and at greater distances than the average. It is possible that stellar H and K might also be sharp in these sharp-lined stars, and could not be differentiated from the interstellar lines by the character or appearance alone. In these uncertain cases the second criterion, the relation of the velocities, must be used. If the difference between the velocities of stellar and interstellar lines is greater than would be expected from their probable errors, then the calcium lines are placed in the interstellar class.

By these criteria a critical examination was made of the 300 odd stars in which H and K were present, to determine whether the calcium

lines were of stellar or interstellar origin. As previously stated there is no ambiguity about the O to B2 stars, and about 175 stars of these types were immediately included in the list of stars with interstellar velocities. In regard to the B3 and B5 stars, which may have stellar H and K, great pains were taken to see that only stars with "interstellar" lines were included. Both of the criteria mentioned above were used and all doubtful cases excluded, so that the 64 B3 and 12 B5 stars placed in the list undoubtedly have "interstellar" lines, and there can be no doubt then that these lines are present in some B3 and B5 stars. A further examination of the B3, B5 stars with the strongest interstellar lines showed that generally they were either Be stars or else absorption stars with unusually strong and well-defined lines—"*c*" stars. Both of these classes are generally believed to be considerably more luminous than the average, and hence at greater distance, thus accounting on Eddington's hypothesis for the strength of the "interstellar" lines.

In this way a list of 261 stars containing "interstellar" calcium was prepared. For 23 of these stars, consisting of 6 Wolf-Rayet stars, 3 novæ, and 14 spectroscopic binaries without reliable γ velocities, only the interstellar velocity was known with sufficient accuracy, thus leaving 238 stars with reliable stellar radial velocities (three of which were later rejected on account of abnormally high velocities) and with reliable velocities of the interstellar cloud in the line of sight of the star. Of these 238 velocities, 228 were obtained at this observatory, leaving only 10 obtained elsewhere, mostly spectroscopic binaries with orbital determinations. It was felt preferable, in order to have a reliable and homogeneous system of velocities, not to include more outside observations, many of which were known to have low weight. While the number of stars with interstellar velocities was thus considerably reduced, it seemed better not to include uncertain values which might distort the results of the analysis.

Analysis of the Interstellar Velocities.

The distribution of these 261 interstellar cloud velocities is represented in Plate 9, the position of each one being plotted on an Aithoff projection perpendicular to the galactic plane. It will be noted that the distribution is far from being symmetrical or uniform, most of the velocities being restricted to about 180° in galactic longitude (from 0° to 180°, with a few scattered stars between 340° and 0° and between 180° and 205°) and within $\pm$ 20° of the galactic equator. Any determination of the solar motion and of the galactic rotation from such an unsymmetrical distribution of velocities will be subject to considerable uncertainty.

The velocity of the Sun with respect to the interstellar clouds will be fairly well determined, as the apex is about longitude 22°, and we have nearly a hemisphere of velocities on one side. But the solar apex will be uncertain, especially in latitude, because of the restriction of the observations to the near neighbourhood of the galactic plane and

their limitation to a hemisphere in longitude. Galactic rotation is rather more favourably situated as it is of double-wave form, depending on the sine of twice the longitude. Hence, with the centre of rotation assumed at 325°, the centre of the globular cluster system, there will be two positive maxima in the velocities at 10° and 190° longitude, and an intervening negative maximum at 100°. Here again we should expect the magnitude of the rotational term to be more closely determined than the direction to the centre.

Hence, it seemed worth while to make a general solution of the 261 interstellar velocities for the solar motion and for the galactic rotation simultaneously. As the velocities are closely confined to the galactic plane, and as the rotational effect can best be referred to the galactic plane, galactic co-ordinates were used in the solution. The usual equations for determining the solar motion were used with the addition of Oort's equations for the galactic rotation. Thus we have the radial velocity V of the interstellar clouds:

$$\mathrm{V} = \mathrm{K} + \cos l \cos b \,.\, x + \sin l \cos b \,.\, y + \sin b \,.\, z + \bar{r}\mathrm{A} \sin 2(l - l_0) \cos^2 b$$

or

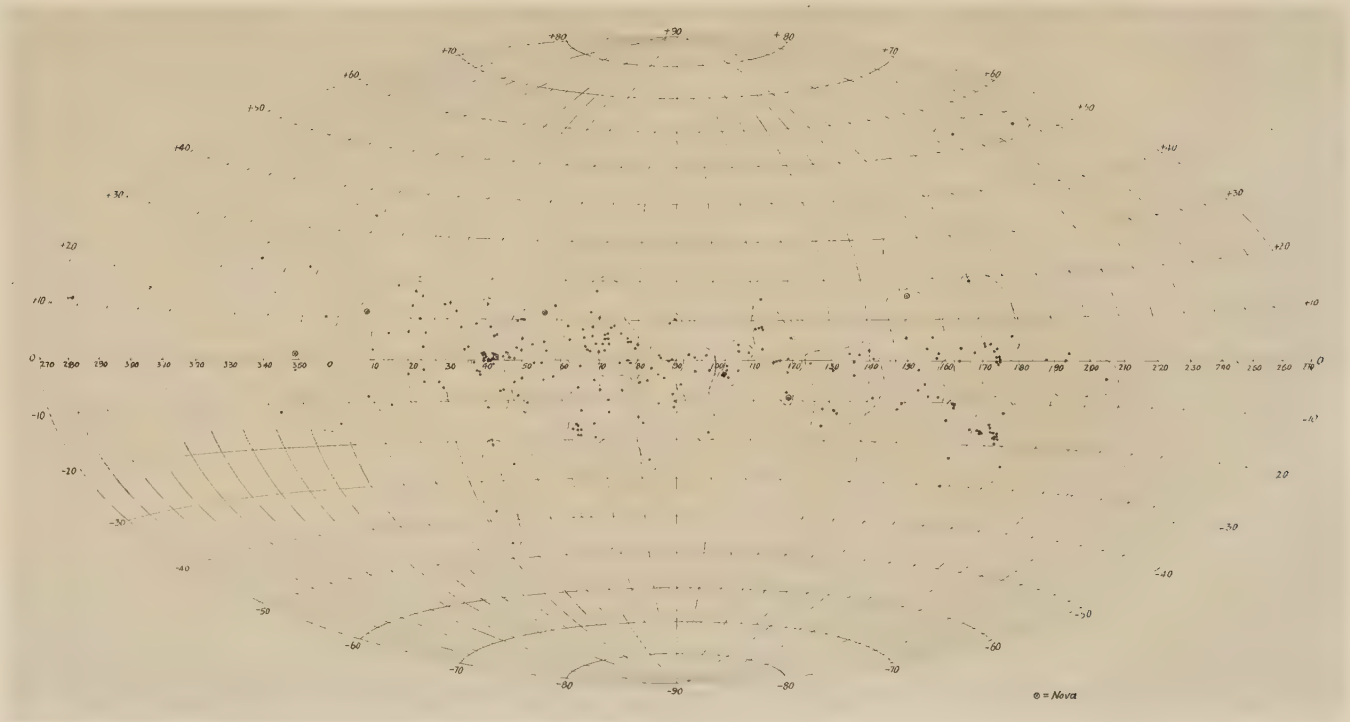
$$\mathrm{V} = \mathrm{K} + ax + by + cz + du + ev,$$

where K has the usual meaning of an average residual velocity, the coefficients a, b, c are obvious and refer to the solar motion, while d, e, u, v have the following values and refer to the galactic rotation, l_0 being the direction to the centre:—

$$d = \sin 2l \cos^2 b.$$
$$e = -\cos 2l \cos^2 b.$$
$$u = \bar{r}\mathrm{A} \cos 2l_0.$$
$$v = \bar{r}\mathrm{A} \sin 2l_0.$$

The usual plan of arranging the stars in a number of longitude groups was followed, and in doing this the plot of the distribution (Plate 9) was useful in arranging them in natural and compact groups. They were finally subdivided into 19 groups, the arrangement being indicated by dotted lines in Plate 9. No attempt was made to divide them into equal areas or equal numbers of stars, but, owing to the clustering of the B stars into groups, these natural divisions were followed as closely as possible. Some idea of the arrangement in longitude and latitude is given in Table II.

Distribution of Calcium Clouds with known Radial Velocities.



Galactic Co-ordinates.

TABLE II.

Arrangement of Groups.

Group.	No.	Region.	Range. Long.	Range. Lat.	$\bar{l}$.	$\bar{b}$.	$\bar{v}$.	$\bar{p}$.
1	4	Oph.	321 to 359	+10 to +35	345·9	+22·2	−12·68	+ 3·35
2	4	Scu.	343 „ 360	+ 2 „ −15	352·6	− 6·8	−12·20	+ 3·88
3	15	Aql., Her.	9 „ 24	−11 „ +18	18·9	+ 5·4	−13·40	+ 5·75
4	8	β Cyg.	28 „ 36	− 8 „ +14	31·7	+ 4·9	−13·14	+ 5·69
5	4	Peg.	34 „ 44	−21 „ −46	38·2	−32·4	− 6·40	+ 5·0
6	32	P Cyg.	37 „ 46	−12 „ +14	41·5	+ 0·6	−13·63	+ 4·15
7	20	α Cyg.	47 „ 58	−11 „ +13	51·6	+ 1·3	−10·83	+ 5·03
8	33	Cep.	58 „ 80	− 1 „ +18	71·0	+ 6·0	−14·96	− 2·32
9	17	Lac.	62 „ 76	− 4 „ −19	66·7	−13·1	−14·50	− 3·20
10	18	β Cas.	80 „ 90	−25 „ +10	85·8	− 5·2	−14·37	− 6·86
11	7	δ Cas.	91 „ 99	−18 „ + 2	95·7	− 3·4	−14·77	−10·53
12	11	h, χ, Per.	102 „ 109	− 4 „ + 4	104·2	− 1·3	−21·39	−18·94
13	15	Cam, Cas.	111 „ 119	− 9 „ +18	115·6	+ 2·5	− 3·51	− 4·39
14	13	Per, Aur.	124 „ 141	−16 „ + 2	131·7	− 4·9	+ 8·50	+ 0·95
15	14	ζ Tau.	148 „ 159	−12 „ +16	153·7	− 1·8	+12·04	− 0·49
16	16	Mon.	164 „ 179	− 1 „ +19	171·8	+ 2·8	+19·03	+ 3·22
17	24	ε Ori.	162 „ 182	−20 „ −10	172·5	−17·2	+19·28	+ 2·22
18	4	CMa.	187 „ 205	− 4 „ + 2	194·2	− 0·3	+31·33	+12·85
19	2	ρ Leo.	204 „ 231	+53 „ +54	217·6	+53·2	+13·40	+ 8·40

All the columns in this table are self-explanatory except the last two, where $\bar{v}$ is the mean radial velocity of the corresponding longitude group and $\bar{p}$ the mean residual or peculiar velocity, the radial velocity with the solar motion removed.

The solution of the normal equations based on observation equations formed from these groups resulted in the following values of the six unknowns :—

$$\begin{aligned} K &= + 0{\cdot}55 \pm 2{\cdot}70 \\ x &= - 17{\cdot}87 \pm 1{\cdot}52 \\ y &= - 8{\cdot}67 \pm 4{\cdot}51 \\ z &= - 1{\cdot}26 \pm 4{\cdot}08 \\ u &= + 4{\cdot}71 \pm 1{\cdot}47 \\ v &= - 5{\cdot}57 \pm 2{\cdot}29, \end{aligned}$$

from which it readily follows that

K = av. residual velocity = + 0·55 ± 2·70 km. per second.
V = solar velocity = 19·90 ± 2·40 „ „
L = longitude of apex = 25°·9 ± 11°·8
B = latitude of apex = + 3°·6 ± 11°·8
$\bar{r}$A = rotational term = + 7·30 ± 1·98 „ „
l_0 = longitude of centre = 335°·1 ± 15°·5.

The solar velocity as determined from the radial velocities of the interstellar clouds comes out practically exactly the same as determined from the naked-eye stars ; a useful and interesting result. The solar apex, however, is displaced about 20° to the eastward from its normal position which, in galactic co-ordinates, is L = 21°·8, B = + 20°·0. The greatest departure is in latitude, which was to be expected on account of the unsuitable distribution of the stars. The K term for the interstellar clouds is practically zero, whereas, as will be seen later, it is about + 4 km. per second from the radial velocities of the same stars.

The rotational term $\bar{r}$A of + 7·3 km. per second is substantial, indicating, if the average value of A from the work of Oort and Plaskett of 0·017 km. per second per parsec be accepted, an average distance of the centre of gravity of the interstellar clouds of some 430 parsecs. The longitude of the centre of rotation, 335°·1, is ten degrees away from the usual value of 325°, but the difference is less than the probable error of the determination.

It will be noted that the probable errors of all the quantities are relatively high, and this has been ascribed by us to the uncertain determination of the solar apex from groups of stars very ill-suited for such a purpose. This general solution has, however, served a good purpose in indicating that the solar velocity with respect to the interstellar clouds is exactly the same as for the naked-eye stars, but it could scarcely be expected that stars so peculiarly distributed as these high-temperature stars, closely confined to the galaxy and extending practically over only some 200° in longitude, should give a determination of the apex of much weight. It seemed desirable, therefore, when the solar velocity agreed with the accepted value, to assume the usual solar motion and carry through a second solution for the two terms of the galactic rotation, the direction to the centre and the magnitude, and for the K term.

The equations thus became

$$K + du + ev = \rho,$$

where the terms on the left-hand side have the same meaning as before and ρ is the mean residual velocity, the radial velocity with the solar motion removed, or $\rho = V - V_0 \cos D$, and is tabulated in the last column of Table II. The results of this solution, using the same groups as before, were

$$\begin{aligned} K &= -0{\cdot}61 \pm 0{\cdot}57 \text{ km. per second.} \\ \bar{r}A &= +7{\cdot}90 \pm 0{\cdot}79 \quad \text{,,} \quad \text{,,} \\ l_0 &= 331^\circ{\cdot}7 \pm 5^\circ{\cdot}7. \end{aligned}$$

It will thus be seen that accepting the usual values for the solar apex has reduced the probable errors to about one-third of their former value and has given a strong value of the galactic rotation for the interstellar clouds, a positive value ten times the probable error, while the direction to the centre agrees with the direction to the centre of the

globular cluster system within its probable error.* The comparison can be seen more directly by tabulating the two solutions.

TABLE III.

Rotation of the Galaxy from the Interstellar Clouds.

	1. General Solution.	2. Particular Solution.
V	$=+19{\cdot}90 \pm 2{\cdot}40$ km. per second.	$+20{\cdot}00$ } Assumed.
L	$=25^\circ{\cdot}9 \pm 11^\circ{\cdot}8$	$21^\circ{\cdot}8$ } Assumed.
B	$=+3^\circ{\cdot}6 \pm 11^\circ{\cdot}8$	$+20^\circ{\cdot}0$ } Assumed.
$\bar{r}$A	$=+7{\cdot}30 \pm 1^\circ{\cdot}98$	$+7{\cdot}90 \pm 0{\cdot}79$
K	$=+0{\cdot}55 \pm 2{\cdot}70$	$-0{\cdot}61 \pm 0{\cdot}57$
l_0	$=335^\circ{\cdot}1 \pm 15^\circ{\cdot}5$	$331^\circ{\cdot}7 \pm 5^\circ{\cdot}7$

The remarkable way in which the motions of the interstellar clouds agree in magnitude and sign with the differential motions that would be produced by a rotation of the galactic system around a distant centre is best shown by a comparison of the observed motions with those computed on the assumption of such a galactic rotation.

TABLE IV.

Comparison of Observed and Computed Velocities.

				1. General Solution.			2. Particular Solution.		
No.	*.	$\bar{l}$.	$\bar{b}$.	$\bar{v}$.	v'.	$\bar{v}-v'$.	$\bar{\rho}$.	ρ'.	$\bar{\rho}-\rho'$.
		°	°						
1	4	346	+22	−12·7	−11·7	− 1·0	+ 3·4	+2·6	+ 0·8
2	4	353	− 7	−12·2	−11·7	− 0·5	+ 3·9	+4·6	− 0·7
3	15	19	+ 5	−13·4	−12·0	− 1·4	+ 5·8	+7·2	− 1·4
4	8	32	+ 5	−13·1	−12·6	− 0·5	+ 5·7	+6·2	− 0·5
5	4	38	−32	− 6·4	−11·0	+ 4·6	+ 5·0	+3·5	+ 1·5
6	32	41	+ 1	−13·6	−13·2	− 0·4	+ 4·2	+4·5	− 0·3
7	20	52	+ 1	−10·8	−14·1	+ 3·3	+ 5·0	+2·1	+ 2·9
8	33	71	+ 6	−15·0	−15·0	− 0·0	− 2·3	−3·1	+ 0·8
9	17	67	−13	−14·5	−14·2	− 0·3	− 3·2	−1·9	− 1·3
10	18	86	− 5	−14·4	−14·0	− 0·4	− 6·9	−6·4	− 0·5
11	7	96	− 3	−14·8	−12·6	− 2·2	−10·5	−7·9	− 2·6
12	11	104	− 1	−21·4	−10·6	−10·8	−18·9	−8·5	−10·4
13	15	116	+ 2	− 3·5	− 6·8	+ 3·3	− 4·4	−8·1	+ 3·7
14	13	132	− 5	+ 8·5	+ 0·8	+ 7·7	+ 1·0	−5·7	+ 6·7
15	14	156	− 2	+12·0	+12·4	− 0·4	− 0·5	−0·1	− 0·4
16	16	172	+ 3	+19·0	+20·9	− 1·9	+ 3·2	+4·5	− 1·3
17	24	173	−17	+19·3	+20·3	− 1·0	+ 2·2	+4·5	− 2·0
18	4	194	0	+31·3	+27·1	+ 4·2	+12·8	+7·2	+ 5·6
19	2	218	+54	+13·4	+12·4	+ 1·0	+ 8·4	+1·4	+ 7·0

* It should be noted that this determination of the direction to the centre of rotation is perpendicular to the direction of Stromberg's axis of asymmetry at galactic longitude 61°.

Comparing the columns $\bar{v}$ and v', observed and computed radial velocities, it is at once seen how closely the observed and computed velocities agree. Except for groups Nos. 12 and 14, which will be referred to later, and for Nos. 5 and 18, which have only four stars each, the agreement is remarkable and generally little exceeds the probable errors of the determination of the velocities. There is apparently little indication of any but very small group or peculiar motions of the clouds, such as have been found in the motions of the B stars themselves, except for the instances noted above.

If the residuals of the radial velocities of the general solution, column $\bar{v} - v'$, are compared with those of the residual or peculiar velocities of the particular solution, column $\bar{\rho} - \rho'$, we find that the principal difference is in the last group, which has probably been partly responsible for giving the discrepant position of the solar apex. The two stars in this group are 54° from the galactic plane, are probably relatively near, and should perhaps not have been used in the solutions.

In the second or particular solution, where the usual solar motion was assumed, and which forms a direct test of the reality of a galactic rotation, a simple comparison of the values of the computed residual or peculiar velocity ρ' with those of the observed $\bar{\rho}$ shows a remarkable correspondence. With the exception of groups 12, 14, 18, and 19, the average residual, $\bar{\rho} - \rho'$, is only 1·3 km. per second. Group 12 is composed of stars confined to the near neighbourhood of the h, χ clusters in Perseus, which are known to be much more distant than the average of the other groups. Similarly, for group 18 all the stars are of O-type, and this group probably is also more distant than the average. The rotational terms, the $\bar{r}$A's, for these two groups would be much greater than for the remaining groups, thus explaining the high residuals in these two cases. No. 19 has been explained above, and that leaves only No. 14, which has a positive residual of 6·7 km. This group is on the boundaries of Perseus and Auriga, where the interstellar lines are rather weak, and hence the distance of the group is probably smaller than the average. This smaller distance, however, could account for only part of the residual of 6·7 km. per second, possibly one-third of it, leaving a difference between the calculated rotational effect and the average velocity of the cloud in this region of 4 or 5 km. per second. This cannot be ascribed to accidental errors, as the interstellar velocities are well determined, the probable error of the mean of the 13 velocities of the group being ±0·8 km. per second. There must hence be a real peculiar or group motion of recession of the interstellar material in this region of about 4 km. per second, but this need cause no surprise when there are known deviations of the same order in the velocities of different parts of the Orion Nebula which is probably a local condensation of the interstellar material.

With the exception of these four groups, whose deviations are thus accounted for, the agreement of the observed velocities with those that would be produced by a rotation of the galaxy is so remarkably close that there can be no reasonable doubt that *this diffuse interstellar matter partakes almost exactly of the differential motions given by a general*

galactic rotation. The smaller differences observed may be reasonably accounted for by small peculiar motions in different parts of the cloud, or, possibly, by the method of grouping employed by which parts of the cloud at greatly different distances may be grouped together.

This important conclusion, which satisfactorily disposes of this first part of the discussion, that relating to the motions of the interstellar clouds, has also a direct bearing on the second part, the distribution of the clouds. For if the cloud follows generally the galactic rotation we may place considerable confidence in the values of the rotation term $\bar{r}$A obtained, which, if the constant A is known accurately, should give accurately the mean distance of the cloud. The mean value of A, as given in the introduction, of ·017 * gives immediately, from $\bar{r}$A = + 7·90 for the cloud, the mean distance of 465 parsecs. But, even if A is not accurately known, the values of $\bar{r}$A for different groupings of stars and clouds should give accurate ratios of the distances of these groupings. If, then, we can determine the rotational term from both the stellar and the interstellar velocities of the same group or groups of stars, we have a reliable method of determining the relative distance of stars and clouds.

II. *The Distribution of the Interstellar Clouds.*

Observational Material.

As the rotational term is to be determined for both stars and clouds, some revision of the data used in the first part is necessary. These referred to interstellar velocities only, and included a number of stars, such as the Wolf-Rayet and the novæ, for which no stellar velocity can be obtained. Also there are some spectroscopic binaries which give reliable interstellar velocities but uncertain stellar, and some constant velocity stars with average interstellar but abnormally high stellar velocities, whose inclusion would distort the mean peculiar velocity of any group to which they belonged. All such objects must be excluded from the revised list, which must only include stars which have reliable stellar and interstellar velocities. Altogether 26 stars were, for these reasons, excluded from the previous list, leaving 235 reliable stellar and interstellar velocities for the new discussion. While all the stars were combined into 19 longitude groups for the general galactic rotation, it would seem preferable to subdivide these into smaller groups, each composed of stars and clouds at approximately the same distances, for the determination of relative distances of stars and clouds by a comparison of their respective rotational terms.

To subdivide the O and B stars into groups according to distance is, however, a very difficult task, as so little is known about the distances of these high temperature stars. The distances are so great that trigonometric parallaxes are worthless, while the method of spectroscopic absolute magnitudes appears to provide little but group parallaxes. It is probable that the dispersion in absolute magnitude in the various groups is much greater than is generally believed, and that such group

* From Oort's value of ·019 and Plaskett's of ·0155 km. per sec. per parsec.

parallaxes do not give very certain information about distances. The mean or statistical parallaxes from proper motions provide information about mean distances, but again the dispersion in absolute magnitude may distort the results in such special circumstances as are considered here. As no better method seemed available, however, and as a first criterion, the 235 stars were divided into different magnitude groups, which should arrange them, roughly at any rate, in order of distance.

Arrangement and Solution of Magnitude Groups.

The number of magnitude divisions should not be too great, or the numbers of stars in the longitude subdivisions will be so small that the solutions will have low weight. Following the method adopted for solving for the galactic rotation from the radial velocities of 875 O to B5 stars, of which the results will be published later, groups including a magnitude difference of one magnitude were used. As there are very few stars brighter than 4·5 magnitude with interstellar lines, the first group comprised all stars brighter than 5·5 magnitude. Many stars in this group are so near, and the number 37 is so small, as to make the solution uncertain, so a second intermediate group of 45 stars from 5·00 to 5·99 inclusive was formed. The third group of 79 stars is from 5·50 to 6·49 magnitude inclusive, and the fourth group of 119 stars includes all 6·50 magnitude and fainter. A fifth group of 69 of the faintest, and probably most distant, of the stars of the fourth group from 7·00 to the faintest, 8·6 magnitude, was also formed. The data about these magnitude groups are combined in Table V.

Table V.

Magnitude Groups.

Group.	Mag. Limits.	No. Stars.	No. Longitude Sub-groups.	Mean Mag.
1	0 to 5·49	37	10	4·41
2	5·00 ,, 5·99	45	8	5·60
3	5·50 ,, 6·49	79	10	6·03
4	6·50 ,, 8·6	119	14	7·08
5	7·00 ,, 8·6	69	11	7·34

These five magnitude groups were each subdivided into longitude groups of the number shown in the fourth column, and each solved for the galactic rotation. The usual solar motion of 20 km. per sec. to the apex at $\alpha = 271°$, $\delta = +28°$ was assumed. Also the direction to the centre of rotation was assumed as 325°, the direction to the centre of the globular cluster system. Under these assumptions the equations reduce to $K + \bar{r}A \sin 2(l - 325°) \cos^2 b = \rho$. The results of $\bar{r}A$ and K for both stars and clouds with their probable errors are given in Table VI.

There are several interesting results of these solutions which may be discussed in turn. Consider the K term first. For the interstellar

clouds it is practically zero, in the mean + 0·12, within the limits of the probable errors for each magnitude group, just as was found in the general solutions. For exactly the same stars which give a zero K term for the interstellar clouds, the stellar K term varies between + 1·4 and + 7·1 in the different magnitude groups. Obviously the K term has nothing to do with the properties of the space concerned but is inherent in the stars themselves. We may add that it has been clearly demonstrated, in the investigation referred to above, that this K term in the O and B stars is due to preponderating peculiar motions of recession of certain special groups of stars, particularly among the brighter stars in the southern hemisphere, which are not balanced by equal motions of approach. Hence the positive K term among the brighter stars, which disappears in the B-type stars at magnitude 6 and fainter. These results, including the galactic rotation for 875 stars, will be published later.

TABLE VI.

Solution of Magnitude Groups.

Group.	$\bar{m}$.	$\bar{r}$A.		K.	
		Stars.	Clouds.	Stars.	Clouds.
1	4·41	+ 1·81±2·83	+ 3·85±1·22	+7·12±1·42	+0·02±0·61
2	5·60	+10·26±2·12	+ 5·02±1·24	+3·99±0·96	+0·97±0·56
3	6·03	+13·86±1·75	+ 7·66±0·90	+1·67±0·77	+0·09±0·40
4	7·08	+16·58±2·20	+ 8·31±1·36	+1·39±1·46	−0·69±0·90
5	7·34	+20·49±2·32	+10·08±1·57	+2·34±0·86	+0·23±0·56

The rotational term $\bar{r}$A is, as will be noted, indeterminate for the small group of the brightest stars. This is partly due to the small numbers and the great part any large peculiar motions would have among so few stars, and perhaps, partly, to the fact that these stars are relatively near and the rotational term small. It should be noted, however, that $\bar{r}$A for the interstellar clouds in this group is fairly well determined, its probable error less than one-third the value, an indication of the smallness of the peculiar motions in the clouds, and that its magnitude, + 3·85 km. per second, is, considering the magnitude and distance, in line with the other values of $\bar{r}$A for the interstellar clouds. The great difference between the probable errors of star and cloud determination in all groups is undoubtedly mainly due to the large part the peculiar motions of the stars play in the case of small numbers and the smallness of such peculiar motions in the clouds.

Considering then the $\bar{r}$A terms in the remaining four groups, composed as already stated of the variable part $\bar{r}$, the average distance of the group of stars or clouds, and the constant part A previously taken as 0·017 kilometres per second, and corresponding to the rotational effect per parsec of distance, we find an increase in $\bar{r}$A and hence in $\bar{r}$, the mean distance, as the magnitude increases. This increase in distance corresponds closely to the increase in faintness at least for the two

extreme groups, No. 2 and No. 5, where $\bar{r}A$ in No. 5 is twice that in No. 2, corresponding to a difference in brightness of four times, which is not greatly different from the ratio of magnitudes, 5·60 to 7·34. While the other ratios are not exact they correspond approximately, so that we can, from the galactic rotation, confirm the relation previously postulated that, generally speaking, the fainter B and O stars are farther away than the brighter. The above relation would place 5·0 magnitude O's and early B's at a distance of about 450, 6·0 magnitude at about 800, and 7·0 magnitude at about 1000 parsecs.

But the most important result of these solutions is undoubtedly the ratio of the rotational term $\bar{r}A$, for the stars of any magnitude group, to that for the interstellar clouds of the same group, the results for the clouds being based on the same stars as the results for the stars themselves. Omitting the first group, which is indeterminate, these ratios for the second, third, fourth, and fifth groups are respectively 2·04, 1·81, 1·99, 2·03, or in the mean 1·97. As A is a constant, the $\bar{r}$, the mean distance of the stars, is almost exactly twice that of the interstellar clouds. This is true for stars of the last four groups at approximately 600, 800, 1000, and 1200 parsecs away, and is the more remarkable when the relatively small numbers of stars in the groups and the effect any peculiar motions of stars or clouds might have on the results is considered. The only possible deduction from these results is that the effective distance at which the interstellar absorption is produced is half the distance of the corresponding stars. This surely means, when groups of stars at such greatly differing distances are involved, *that the interstellar matter is uniformly distributed*, and is a striking confirmation of Eddington's hypothesis.

Intensity of the "Interstellar" Lines.

In view of Struve's interesting results on the relation of the intensity of the interstellar lines to the apparent magnitude, and hence probably to the distance, of the stars, it seemed worth while to re-group these stars according to the relative intensity of the K line, and again solve for the galactic rotation and the distribution of the interstellar clouds.

The accurate estimation of the intensity of the K line, which is about the practical limit of visibility in the spectrum owing to absorption of the glass in the optical train, is no easy matter and appears to be beset with many pitfalls. While Struve's method of visual estimates is probably as satisfactory as can be expected from the character of the material, there is no assurance that one's mental scale would remain constant, and it seemed best to us to compare every intensity with some fixed standard. Fortunately such a standard was already available in one of the scales used by Young and Harper for estimating the relative intensities of the lines used in spectroscopic parallax work. One of these scales was especially suitable as it contained ten artificial lines, ranging in intensity from practically clear glass, called intensity 10, to a faint line just distinguishable from the continuous background, called intensity 1. The lines were hard and sharp, almost identical in

appearance with the " interstellar " lines, which made the comparison more easy and accurate.

The main difficulty in the estimates is the great variation in the intensity of the continuous background in star spectra of various degrees of exposure, and the difficulty of matching the intensity of the lines in star backgrounds of varying strength with the artificial lines in a background of constant strength. Fortunately we had a large number of spectra of varying exposure for each of a considerable number of stars, and it was found that there was a continuous increase in the estimated intensity of the K line with the increase in the strength of the continuous background. A careful comparison and matching of intensities with the standard for spectra of different density enabled us to obtain a fairly definite and constant relation between the intensity and the strength of the continuous spectrum. This strength was defined by the wave-length to which the continuous spectrum extended,

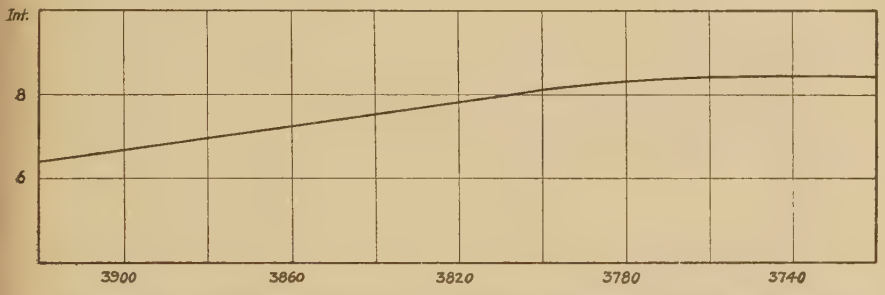


Fig. 1. Differences in Estimated Intensity of K for Different Extensions of the Continuous Spectrum.

which varied between about λ 3700 and λ 3920. No reliable estimates of the intensity of K could be made unless the continuous spectrum extended at least to λ 3920.

It was found that the estimated intensity of K increased slowly with the intensity of the continuous background, with the extension into the violet. The amount of the increase is represented by the curve in fig. 1, which is a mean curve based upon the measures of numerous spectra of several stars. It is an inclined straight line, the intensity increasing about 0·3 unit for each 10A of the extension of the spectrum into the violet until about λ 3800, when it curves towards the horizontal.

In estimating the intensity of the K line in some 600 spectra of about 230 stars, the standard scale was placed in contact with the spectrum to be estimated and shifted along under an eye-piece until the K line was adjacent to the artificial lines, when it was easy to select the artificial most closely matching the star line. The number of this artificial line, with the wave-length to which the spectrum extended, was noted. After the intensity of K had thus been estimated on the 600 plates, all these estimates were reduced by means of the curve in fig. 1 to the intensity of a spectrum in which the background

extended to λ 3860, the average extension of the continuous background, and thus made directly comparable with each other. The individual corrected estimates were averaged for each star, and these means were then used for regrouping the stars into intensity instead of magnitude groups.

Before giving the details and solutions of these groups it will be worth while to attempt to translate the arbitrary scale of estimates from 1 to 10 into magnitude differences. Unfortunately, we have no record of how the scale was formed, although even this would be of little use, as the light used was probably daylight of quite a different wave-length to that of the K line and with a different characteristic curve. Again, we have sensitometer patches on each spectrum plate, but these have the same drawback of being made with an incandescent lamp with maximum effect around λ 4700 instead of λ 3930. Consequently, a series of exposures was made on the star 9 Cam., with strong interstellar lines, and a comparison of these by the microphotometer with one another and with sensitometer patches, made with both violet light and light from the usual source, showed the following relation between the successive numbers of the artificial scale. These relations are, of course, only approximate, but give some idea of the intensity of the absorption corresponding to the numbers of the scale. The magnitude differences between the continuous background of the standard and the corresponding numbers of the lines are here given:

Line .	10	9	8	7	6	5	4	3	2	1
Mag. Diff.	·90	·72	·55	·43	·33	·28	·23	·20	·17	·14

It will be noted that the steps are much greater for the strong than for the weak lines, a result of the way in which the scale was made. A comparison of the intensity of the interstellar line K in 9 Cam. with that of the continuous background by means of successive exposures of different lengths showed a difference of intensity of nearly one magnitude. These results agree approximately with those of Struve for the intensity of K, but the ratio for the steps most frequently used here is slightly greater than that assumed by him.

Arrangement in "Intensity" Groups.

The intensities of the "interstellar" K line varied from 1·6 in the O8 star ι Orionis to 9·5 in H.D. 14134, a B2 star of 6·66 magnitude in the h, χ clusters in Perseus. There are only 14 stars with intensities less than 4·4, whose mean apparent magnitude is 4·85 and mean type is B1·6. The number of stars out of the 235 used in the last grouping for which intensities of the "interstellar" K line could be reliably estimated was 226, of which 122, or 54 per cent., had intensities greater than 7·0.

The stars were arranged in the order of intensity of K and divided first of all into four intensity groups, Nos. 1, 2, 4, 5, of the following table, which gives all the useful data about the groups. Then, owing to the uncertainty attaching to the results from the nearer stars, in

this case presumably those with low K intensities, all stars with intensities of K less than 4·4, where there is a natural break in the run of the intensities, were discarded, and the remaining 28 stars, too small a number to solve separately, were combined with those of group 2 to form group 3 of 90 stars.

TABLE VII.

Arrangement in Intensity of K Groups.

Group.	No. Stars.	No. Long. Groups.	Range of Intensity.	Mean Int.	Mean Mag.	Mean Type.
1	42	9	0 to 5·9	4·72	5·77	B2·2
2	62	10	6·0 „ 6·9	6·50	6·45	B1·2
3	90	11	4·4 „ 6·9	6·08	6·35	B1·6
4	79	11	7·0 „ 7·9	7·46	6·73	Bo·9
5	43	9	8·0 „ 9·5	8·42	6·56	Bo·4

It is interesting to note that, although there is a change of the mean intensity of K from 4·72 to 8·42, the increase of mean apparent magnitude is only from 5·77 to 6·56, and of temperature from B2·2 to Bo·4. Also, in the four last groups, with a change in intensity of K from 6·08 to 8·42 there is a change of less than 0·4 magnitude in the mean magnitude. In the hope of providing some explanation, the stars were divided in the four main groups 1, 2, 4, 5 into three type divisions, O, Bo to B2, and B3 to B5, and the number of the stars of each type, with the mean magnitude and type, are given in Table VIII.

TABLE VIII.

Arrangement in Types.

Group.	Intensity Range.	O Stars.			Bo to B2 Stars.			B3, B5 Stars.		
		No.	Type.	Mag.	No.	Type.	Mag.	No.	Type.	Mag.
1	0 to 5·9	3	07·8	4·47	14	B1·1	5·37	25	B3·3	6·38
2	6·0 „ 6·9	15	06·7	6·39	25	B1·3	6·38	22	B3·1	6·58
4	7·0 „ 7·9	22	06·8	6·78	38	B1·3	6·48	19	B3·1	6·49
5	8·0 „ 9·5	14	07·1	6·08	22	B1·2	6·69	7	B3·3	6·48

But this does not give much additional information except that the mean magnitudes of all types, especially of the three last groups, are very similar.

There is, however, a very considerable difference in the average distances of these groups, as will appear from the solutions below, and the only possible explanation seems to be a larger dispersion in absolute magnitude in any one type than is generally believed. It will be noted that the mean magnitude of group 5 of Table VII. is less than that of group 4, and yet, as the solutions show, the former is at nearly twice the distance. This cannot be accounted for by the change in type of half a division, and must be due to some other cause. On examining

the stars of group 5, it was found that the smaller mean apparent magnitude was due not to the smaller magnitude of all the stars, but to the presence of a number of relatively bright stars. Thus 11 of the stars have a mean apparent magnitude of 5·61 as compared with 6·90 for the remaining 32 stars. These brighter stars include such well-known super-giants as H.D. 1337 and B.D. 6° 1309, and many with sharp, strong, stellar lines, all probably being stars of great intrinsic luminosity.

The presence of a number of super-giant stars in increasing proportions with increase in the intensity of the K line and in distance, together with the advance in spectral type from B1·6 to B0·4 for groups 2 to 5, and consequent increase in absolute magnitude, may account for the nearly constant apparent magnitude in these groups. In any event there seems to be no doubt of the presence of stars of very great luminosity, instanced by several B3 and B5 stars brighter than 6th magnitude in the last group, combined with the usual dispersion in absolute magnitude.

Solution of Intensity Groups.

After the 226 stars, of which there were plates with K lines measured for radial velocity and estimated for intensity, had been arranged into intensity groups as in the above table (Table VII.), each intensity group was divided into the number of longitude groups shown in the third column of the table.

These groups were then solved for the galactic rotation and the K term for both stellar and cloud velocities, separately, the usual solar motion and the longitude of the centre of rotation being assumed as in the previous magnitude groups. The results of these solutions, with their probable errors, are as shown in the following table :—

Table IX.

Solutions for the Intensity Groups.

Group.	Mean Int.	No. Stars.	$\bar{r}$A.		K.	
			Stars.	Clouds.	Stars.	Clouds.
1	4·72	42	+ 3·64±3·25	+ 4·97±0·77	+6·12±2·14	+1·15±0·50
2	6·50	62	+12·12±1·88	+ 4·93±0·92	+4·66±1·27	+0·10±0·62
3	6·08	90	+10·22±1·72	+ 5·03±0·85	+5·24±1·17	+0·10±0·58
4	7·46	79	+14·53±2·93	+ 6·91±1·06	−0·14±1·95	−0·21±0·70
5	8·42	43	+27·52±2·46	+13·72±1·16	+3·18±1·74	−1·17±0·82

Again, the K term for the interstellar clouds is negligibly small, the mean of the five values being 0, while it ranges up to 6·1 for the stars. When it is remembered that, for the magnitude groupings as well as for the general solutions, the value was of the same order, there can be no doubt that there is no K term for the interstellar clouds, though it has a high positive value for the stars.

The value of $\bar{r}$A is indeterminate for the stars of the first group, owing probably, as previously, to large peculiar motions in the velocities of these nearer stars. The $\bar{r}$A for the clouds is, however, reliable, probable error less than $\frac{1}{6}$ value, as any peculiar motions here are small, and is practically identical for groups 1, 2, 3, indicating possibly some uncertainties in the estimates of the intensities of K in those stars in which it is weaker, and some mixing up of nearer stars with more distant ones. It was to avoid this difficulty that group 3 was formed of groups 1 and 2, omitting 14 stars with intensities of K less than 4·4 which were relatively close to us and hence would give uncertain values. Moreover, in this combined group there are about twice as many stars as in group 1, and one and a half times as many as in group 2.

In order to show how closely the residual velocities of stars and clouds follow those that would be produced by a rotation of the galaxy around a distant centre at $\lambda = 325°$, the resulting residual velocities for both stars and clouds were computed for the various values of $\bar{r}$A and K in the last three magnitude groups (Table VI.) and the last three intensity groups (Table IX.). The groups of low magnitude and low intensity are not included on account of the relative nearness of stars and clouds in both groupings, and the uncertainty, at least for the stars, of $\bar{r}$A.

In order to enable the comparisons to be more readily made, the corresponding magnitude and intensity groups are placed adjacent to one another—in one table (Table X.) for the stars, and in another (Table XI.) for the clouds—giving in each the number of the longitude group, the number of stars, and the mean longitude, as well as the comparison of residual velocities. As the mean longitudes are by no means the same in the different groupings, those which most nearly agree are placed in the same line so as to facilitate comparison between the observed and computed velocities. It will be found that such comparisons are worth while, in order to show how closely the stars and clouds follow the double wave-form of the galactic rotation, and how small are the differences, considering the effect of peculiar motions in such small numbers of stars, and considering also the possible effects of group motions in the B stars, such group motions being probable in stars which are arranged into groups in position.

Let us compare first of all the computed with the observed stellar residual velocities, the velocities with the solar motion removed (Table X.). It will be noted immediately, as the eye glances down the columns, the general swing from positive velocities to negative and back again to positive with the progression in longitude, obviously due to the double wave-term of the galactic rotation, in each of the six magnitude and intensity groups. Generally speaking, the differences between observed and computed velocities are relatively small. The larger differences, except around galactic longitude 100°, where the very distant Perseus clusters distort the results, are mostly in the longitude sub-groups where the numbers of stars are small, and consequently where the peculiar velocities play a relatively large part.

TABLE X.

Comparison of Residual Stellar Velocities in Magnitude and Intensity Groups.

Magnitude 5·5 to 6·5.

$\bar{r}A = +13{\cdot}86$. $K = +1{\cdot}67$.

Group.	No.	Mean Long.	Residual Velocities. Observed.	Computed.
		°		
1	3	351	+22·8	+12·2
2	5	21	+ 9·6	+14·5
3	11	33	+10·4	+11·2
4	11	48	+ 4·6	+ 5·2
5	16	67	− 2·6	− 4·0
6	9	81	−13·7	− 9·2
7	3	134	+ 8·0	− 3·5
8	8	158	+ 7·8	+ 7·6
9	11	175	+ 9·9	+13·4
10	2	192	+29·1	+15·5

Magnitude 6·5 to 8·6.

$\bar{r}A = +16{\cdot}58$. $K = +1{\cdot}39$.

		°		
1	2	351	+36·6	+14·2
2	8	18	+ 1·6	+17·6
3	16	41	+11·8	+ 9·7
4	11	51	+ 3·8	+ 4·0
5	11	66	− 1·3	− 4·5
6	11	76	− 7·8	− 7·8
7	14	91	−13·6	−14·1
8	8	103	−40·0	−14·8
9	13	113	− 5·9	−13·2
10	8	134	+ 3·1	− 4·5
11	5	153	+ 5·7	+ 6·5
12	8	173	+19·3	+15·2
13	3	175	+ 8·3	+14·7
14	1	231	+17·0	+ 2·3

Magnitude 7·0 to 8·6.

$\bar{r}A = +20{\cdot}49$. $K = +2{\cdot}34$.

		°		
1	15	40	+12·5	+12·5
2	7	51	+ 3·3	+ 5·4
3	4	66	− 0·6	− 5·6
4	3	76	−14·0	−11·3
5	12	85	−16·4	−15·3
6	4	104	−38·6	−17·9
7	7	113	− 3·7	−15·9
8	5	136	− 0·7	− 3·6
9	3	155	+ 7·6	+ 9·1
10	8	172	+17·9	+19·1
11	1	231	+17·0	+ 4·0

Intensity 4·4 to 6·9.

$\bar{r}A = +10{\cdot}22$. $K = +4{\cdot}24$.

Group.	No.	Mean Long.	Residual Velocities. Observed.	Computed.
		°		
1	2	345	+36·2	+10·8
2	2	17	+ 5·9	+14·0
3	14	38	+10·4	+ 9·9
4	11	61	+ 6·4	+ 2·1
5	13	75	− 7·3	− 2·4
6	8	89	− 6·8	− 5·3
7	8	131	− 2·1	− 5·0
8	6	131	+ 4·5	− 0·5
9	8	156	+ 7·0	+ 8·0
10	13	174	+14·4	+12·6
11	3	205	− 2·6	+ 9·8

Intensity 7·0 to 7·9.

$\bar{r}A = +14{\cdot}53$. $K = -0{\cdot}14$.

		°		
1	8	12	− 5·5	+14·3
2	12	38	+ 8·9	+ 8·0
3	11	51	+ 7·2	+ 1·7
4	10	65	− 1·5	− 5·3
5	4	75	− 7·8	− 9·5
6	8	89	−20·2	−13·7
7	3	110	−23·9	−13·9
8	3	139	+ 1·7	− 3·2
9	9	160	+10·6	+ 7·2
10	10	174	+17·2	+12·2
11	1	192	+44·6	+14·4

Intensity 8·0 to 9·5.

$\bar{r}A = +27{\cdot}52$. $K = +3{\cdot}18$.

		°		
1	2	358	+35·7	+24·3
2	2	17	+12·2	+29·5
3	6	34	+21·5	+21·3
4	2	47	+11·8	+10·4
5	12	70	− 3·4	−10·4
6	4	97	−26·8	−23·4
7	9	107	−28·4	−23·5
8	4	141	− 4·1	− 0·9
9	2	169	+13·6	+16·0

TABLE XI.

Comparison of Residual Cloud Velocities in Magnitude and Intensity Groups.

Magnitude 5·5 to 6·5.

$\bar{r}A = 7{\cdot}66$. $K = +0{\cdot}09$.

Group.	No.	Mean Long.	Residual Velocities. Observed.	Computed.
		°		
1	3	351	+ 5·0	+5·9
2	5	21	+ 9·0	+7·2
3	11	33	+ 4·7	+5·3
4	11	48	+ 5·4	+2·1
5	16	67	− 2·4	−3·0
6	9	81	− 7·2	−5·9
7	3	134	− 1·7	−2·8
8	8	158	− 1·1	+3·4
9	11	175	− 5·6	+6·6
10	2	192	+12·4	+7·7

Magnitude 6·5 to 8·6.

$\bar{r}A = +8{\cdot}31$. $K = -0{\cdot}69$.

		°		
1	2	351	+ 1·1	+6·7
2	8	18	+ 4·2	+7·4
3	16	41	+ 6·2	+3·4
4	11	51	+ 4·7	+0·6
5	11	66	− 4·4	−3·7
6	11	76	− 3·2	−6·2
7	14	91	− 7·2	−8·5
8	8	103	−23·1	−8·8
9	13	113	− 6·2	−8·0
10	8	134	+ 0·3	−3·7
11	5	153	− 1·6	+1·8
12	8	173	+ 3·9	+6·2
13	3	175	− 3·2	+5·9
14	1	231	+14·2	−0·2

Magnitude 7·0 to 8·6.

$\bar{r}A = +10{\cdot}08$. $K = +0{\cdot}23$.

		°		
1	15	40	+ 6·4	+5·2
2	7	51	+ 6·0	+1·7
3	4	66	− 1·0	−3·7
4	3	76	− 3·4	−6·5
5	12	85	− 8·5	−8·5
6	4	104	−21·8	−9·8
7	7	113	− 8·2	−8·8
8	5	136	+ 1·8	−2·6
9	3	155	− 0·8	+3·1
10	8	172	+ 2·7	+8·0
11	1	231	+14·2	+1·1

Intensity 4·4 to 6·9.

$\bar{r}A = +5{\cdot}03$. $K = +0{\cdot}10$.

Group.	No.	Mean Long.	Residual Velocities. Observed.	Computed.
		°		
1	2	345	− 1·2	+ 3·3
2	2	17	+ 4·9	+ 4·9
3	14	38	+ 5·7	+ 2·9
4	11	61	− 0·4	− 1·0
5	13	75	− 0·4	− 3·2
6	8	89	− 6·0	− 0·6
7	8	113	− 6·9	− 4·4
8	6	131	+ 2·7	− 2·2
9	8	156	− 1·7	+ 2·0
10	13	174	+ 4·9	+ 4·1
11	3	205	+ 3·4	+ 0·2

Intensity 7·0 to 7·9.

$\bar{r}A = +6{\cdot}91$. $K = -0{\cdot}21$.

		°		
1	8	12	+ 4·6	+ 6·7
2	12	38	+ 6·3	+ 3·7
3	11	51	+ 4·8	+ 0·6
4	10	65	− 0·9	− 2·6
5	4	75	− 2·4	− 4·7
6	8	89	−10·6	− 6·7
7	3	110	− 9·7	− 6·7
8	3	139	− 0·3	− 1·7
9	9	160	+ 0·2	+ 3·3
10	10	174	+ 2·7	+ 5·6
11	1	192	+13·0	+ 6·7

Intensity 8·0 to 9·5.

$\bar{r}A = +13{\cdot}72$. $K = -1{\cdot}17$.

		°		
1	2	358	+ 8·2	+ 9·4
2	2	17	+ 6·1	+11·9
3	6	34	+ 7·5	+ 7·9
4	2	47	+ 6·6	+ 2·4
5	12	70	− 5·4	− 8·0
6	4	97	−15·5	−14·3
7	9	107	−17·7	−14·5
8	4	141	+ 1·2	− 3·2
9	2	169	+ 1·9	+ 5·3

The high negative velocities of the stars in the neighbourhood of the h, χ clusters in Perseus give high negative differences in the second and third magnitude groups—note sub-group 8 in second, and sub-group 6 in third (Table X.). That this is due to the much greater distance of this group than the average of the other groups is obvious when we compare the above magnitude groups with the group of highest intensities, 8·0 to 9·5. It will be noted that the differences above of some − 20 km. have in this case practically disappeared (difference 3·4 in sub-group 6 and 4·9 in 7). This appears to show clearly that the method of grouping by the intensity of the "interstellar" K line arranges the stars much more closely in order of distance than grouping by magnitudes. The average distance of the stars in this intensity group $\bar{r}$A/A is about 1600 parsecs, and, as the residuals in Perseus are still negative, the Perseus group is indicated to be about 2000 parsecs distant, which does not differ materially from the slightly larger estimates from other sources.

It will be noted that the differences between observed and computed are generally smaller in the intensity than in the magnitude groups. This is an indication that the grouping according to intensity arranges the stars into groups of more nearly equal distances than the grouping in magnitudes, in which, therefore, the dispersion in absolute magnitudes must be considerable. This is particularly noteworthy in the group of only 43 stars of intensity 8·0 to 9·5, where the correspondence is remarkably close, indicating that the stars of the group are all at approximately the same distance of about 1600 parsecs. Nobody who compares the observed and computed velocities in this group can fail to be convinced of the reality of a rotation of the galaxy around a distant centre at longitude 325°.

The same is true of all the groups of Table X., which show markedly that, if the distribution of velocities with longitude is not due to a rotation of the galaxy, the cause is something which produces a velocity distribution almost identical with that which would be produced by a rotation of the galaxy. The same thing is shown, though even more convincingly, by a comparison of the residual velocities of the interstellar clouds in Table XI., where the peculiar motions of the clouds are much less than those of the stars and the agreement is correspondingly closer than with the stars. Any large deviations here are probably due to the large dispersion in distance in the groups caused by the uncertainty of the methods employed for defining the distances. It seems to us that no one can compare the observed and computed velocities in Tables X. and XI., and also in Table IV., without being convinced that the motions of the O and early B stars, and also, more particularly, of the interstellar clouds, confirm in a remarkable degree the hypothesis of a rotation of the galaxy.

Turning again to the results of the solutions of the intensity groups (Table IX.) we at once obtain as before from the $\bar{r}$A of groups 3, 4, 5 that the average distances of the stars in these groups are 600, 850, 1600 parsecs; the first two somewhat similar to the magnitude groups, but the last considerably further away than the group of stars fainter

than 7·0 magnitude. As indicated above, this shows that intensity estimates of the K line are more critical in differentiating distance than magnitude differences.

Finally, comparing the $\bar{r}$A for stars and clouds in the three major groups 3, 4, 5 of Table IX., the remarkable relation is again found that its value for the stars is almost exactly double that for the clouds. The exact ratios are 2·03, 2·10, 2·01—in the mean 2·05 as compared with the mean of 1·97 for the magnitude groups. This gives a final mean for magnitude and intensity groups of 2·01, or the average distance of the stars is practically exactly twice that of the clouds, or the centre of gravity of the absorbing matter producing the " interstellar " calcium lines is exactly half the distance of the stars in whose spectra these lines appear. When we consider that this relation holds for star groups of distances varying between 600 and 1600 parsecs, there can be no possible doubt that *Eddington's hypothesis of uniform distribution of the interstellar matter is fully confirmed.*

Conclusions.

The conclusions that may be legitimately drawn from this investigation may be briefly summarized.

1. The general solution simultaneously for the solar motion, the K term, and the galactic rotation of the interstellar clouds from the Ca^+ velocities of 261 stars resulted in a solar velocity of 19·9 km. per second, the same as the usual accepted velocity of 20 km. per second, to an apex about 20° away from the normal apex; this discrepancy being probably due to the unsymmetrical and unsuitable distribution of the stars. The K term is zero within its probable error, while the rotational term of + 7·3 km., equivalent to a mean distance of the clouds of over 400 parsecs, is directed to a centre at galactic longitude 335°·1, 10° away from its usual position of 325°.

2. As the stars were poorly distributed for the determination of the solar apex, a second solution assuming the usual solar motion was carried through for the K term and the galactic rotation. The K term is again zero, the rotational term $-$ 7·90$\pm$0·79 km. per second, and the galactic centre at longitude 331°·7$\pm$5°·7. A comparison of the observed mean velocities of the groups with those computed from the above values showed such remarkable correspondence as strongly to confirm the theory of a rotation of the galaxy.

3. The 235 stars, for which both stellar and interstellar velocities were known, were divided into five magnitude groups, and consequently approximately into five groups at different distances, and again solved for the K term and the galactic rotation. This solution has the interesting and important result that for all except the group of brightest and nearest stars the rotational term for the stars is almost exactly twice that for the clouds, or the stars are at twice the mean distance of the clouds.

4. The intensity of the " interstellar " K line was estimated in the spectra of 226 stars, for which there were both stellar and cloud velocities.

The stars were again arranged into five groups according to the intensity of K, and consequently, on Eddington's theory, according to the distance. A solution for the K term and the galactic rotation again showed for the three groups with the strongest K lines, the remarkable relation that the rotational term for the stars is almost exactly twice that for the clouds.

5. A comparison of the mean observed velocities of both stars and clouds in the longitude divisions of three magnitude and three intensity groups showed such a close agreement with the computed velocities as to leave no reasonable doubt of the reality of differential motions of these high temperature stars and the intervening clouds, corresponding almost exactly with those that would be produced by a rotation of the galactic system around a distant massive centre in the same direction as the centre of the globular cluster system.

6. The relations found in 3 and 4 above, that the rotational term for the stars is almost exactly twice that for the clouds in each of four different magnitude groups and three different intensity groups, can only mean that the average distance of each of seven different groupings of stars is exactly twice that of the average distance of the corresponding groups of clouds. When it is considered that this relation holds for groups of stars of very different distances distributed over a considerable portion of the galactic plane, the only possible conclusion that can be drawn from these relations provides a strong confirmation of Eddington's hypothesis of a uniform distribution of the interstellar material.

The Shortt Clocks at the Royal Observatory, Greenwich, with Special Reference to the Effect of the Variation of Arc. By J. Jackson, M.A., D.Sc., and W. Bowyer.

(*Communicated by the Astronomer Royal.*)

Introduction and Summary.—The performance of the two clocks, Shortt 3 and Shortt 11, up till the end of 1928 has been given in two papers published in the *Monthly Notices* for 1928 March and 1929 January. Shortt 3 continued to follow pretty closely the formula based on the performance in 1927 up to 1929 May, but afterwards it showed rather sudden changes of rate of the order of $0^{s}\cdot02$ per day, lasting for about a week. These irregularities took place at intervals up till the end of October, when the clock case was opened. The irregularities in the clock rate were accompanied by irregularities in the arc, and an examination of the residuals indicated that changes in the arc produced the theoretical effect calculated for a simple pendulum. Re-examination of the residuals from the formula in 1928 showed that they could be practically explained by the observed variations in the arc. The principal change in 1928 was a decrease in the semi-amplitude by about 0·1 mm. or $18''$ in the middle of July. It then seemed worth while to reconsider the performance of the clock from the time of its

erection. In the period 1925 January till 1926 June the case was leaking and the arc varying. It was found that the changes of rate could be largely explained by:

(1) Theoretical change of rate with change of arc.
(2) Change of daily rate of 0·606 seconds for each change of 1 inch of mercury in the pressure.
(3) Gradually decreasing secular acceleration in the losing rate.

The apparent sudden reduction of the acceleration in the autumn of 1925 is explained by the changes in pressure and arc.

An interesting result is that a reduction in the pressure leads to such an increase in the arc that the reduction of the pressure below three-quarters of an inch of mercury increases the period of vibration. The changes of arc and rate with change of pressure are shown in fig. 2.

Various possible causes have been assigned to the secular change in the period of the pendulum. It has been shown that the change is many times larger than that due to the normal growth of invar, the material of which the pendulum is constructed. It has been suggested that it might be due to a sinking of the bob, which is of type metal. A new bob of invar was prepared to replace the old one, and was fitted to the pendulum in 1929 November. It is considerably larger than the old bob, and weighs 22 lbs. against 14 lbs. So far as the observations go there is no certain sign of any secular change, although sufficient time has not yet elapsed for definite conclusions to be drawn.

The conclusion to be drawn from this paper is that hopes of checking the regularity of the earth's rotation have considerably increased. The variation in the amplitude of vibration when the case is airtight has not been explained, although it is well within the limits which the makers guaranteed. It may be that the residual changes in arc and in rate are associated with slight temperature changes. The control temperature has been kept rather low, so that in summer heat may be leaking into the clock room instead of out of it. The clock is firmly fixed with metallic connections to the wall, and the temperature in the clock case may be slightly different from that of the room.

It will be necessary when an attempt is made to check the uniformity of the earth's rotation to make a record of the arc and allow for it in the rate of the clock. This can be easily done within the required limits. It is essential to have the clock case perfectly sealed, as the principal irregularities in Shortt 3 for the first eighteen months were undoubtedly due to change in pressure accompanied by change in arc. The improved performance of both Shortt 3 and Shortt 11 since 1926 July is to be attributed to the perfect sealing of both cases at that time.

Shortt 3.—A comparison of the observed clock errors with those predicted by formula (3) of *M.N.*, **88**, 469, is given in column 3 of Table I. This is a continuation of Table I. of *M.N.*, **89**, 242. The standard temperature to which the formula was brought forward was 62°·8 F. At the beginning of the year the temperature as registered by the thermograph was only very slightly (about 0°·1) below this standard. In May and June it was practically standard, in July 0°·3

above, in August 0°·8 above, in September 1°·5 above, and before the end of October it had fallen to normal.

TABLE I.

Comparison of Clock Errors Observed with Transit B with Formula (3), *M.N.*, **88**, 469 *for Shortt* 3, *and with Formula* (6), *M.N.*, **89**, 243, *for Shortt* 11. *Continuation of Table I., M.N.*, **89**, 242, *and Table II., M.N.*, **89**, 244.

1929.	Temp.* Corr. for Shortt 3.	Shortt 3. Obs.−C_3.	No. of Stars. S, N.	Shortt 11. Obs.−C_6.
	s	s		s
Jan. 12	−·023	−·264	5, 5	−0·163
16	·023	−·323	5, 4	−0·233
20	·023	−·303	5, 4	−0·237
Feb. 3	·026	−·202	5, 5	−0·138
7	·026	−·169	5, 3	−0·098
12	·028	−·152	4, 3	−0·052
17	·032	−·195	5, 4	−0·090
21	·034	−·105	5, 4	+0·018
Mar. 1	·037	−·078	5, 3	+0·096
7	·040	−·028	5, 4	+0·143
11	·043	+·002	4, 3	+0·165
17	·046	+·032	4, 3	+0·221
23	·051	+·021	5, 4	+0·267
26	·054	+·019	4, 3	+0·303
30	·058	+·044	5, 3	+0·357
Apr. 3	·062	+·034	5, 4	+0·367
9	·068	+·034	5, 2	+0·430
14	·070	+·047	4, 3	+0·517
21	·074	+·045	4, 4	+0·601
30	·080	+·095	5, 4	+0·788
May 3	·080	+·100	5, 3	+0·827
7	·080	+·097	4, 4	+0·838
19	·083	+·197	5, 3	+0·963
27	·084	+·255	5, 3	+1·082
June 6	·082	+·126	5, 3	+1·124
10	·082	+·178	6, 3	+1·164
11	·082	+·194	3, 2	+1·185
16	·082	+·234	6, 3	+1·237
19	·082	+·281	5, 4	+1·294
24	·081	+·216	6, 3	+1·329
26	·081	+·226	5, 3	+1·350
July 8	·081	+·399	5, 4	+1·527
13	·081	+·447	5, 4	+1·574
15	·080	+·450	5, 3	+1·605

* The zero date from which this is computed is 1928 October 26. To get temperature correction for Shortt 11, multiply that for Shortt 3 by 0·847.

TABLE I.—*continued.*

1929.	Temp.* Corr. for Shortt 3.	Shortt 3. Obs.−C_3.	No. of Stars. S, N.	Shortt 11. Obs.−C_6.
	s	s		s
July 17	−·081	+·497	6, 3	+1·658
21	·079	+·426	5, 3	+1·673
25	·070	+·421	4, 3	+1·767
31	·052	+·416	7, 5	+1·842
Aug. 8	·032	+·510	6, 3	+1·957
12	·024	+·456	6, 3	+1·968
17	−·012	+·544	7, 4	+2·034
25	+·007	+·695	6, 3	+1·995
26	·009	+·714	4, 4	+1·997
29	·017	+·783	4, 4	+2·022
Sept. 1	·027	+·825	7, 5	+2·011
6	·048	+·853	5, 4	+1·989
10	·068	+·802	4, 3	+1·975
14	·090	+·833	4, 3	+2·064
16	·102	+·774	1, 2	+2·027
20	·124	+·745	5, 3	+2·034
23	·139	+·702	5, 4	+2·007
27	·155	+·730	4, 4	+2·064
Oct. 4	·176	+·691	4, 3	+1·996
6	·180	+·672	5, 4	+1·914
11	·186	+·771	6, 3	+1·904
14	·188	+·774	6, 4	+1·833
19	·191	+·908	5, 4	+1·861
23	·193	+·968	4, 2	+1·826
26	·193	+·974	5, 4	+1·756
31	·193	..	5, 4	+1·725
Nov. 3	·193	..	5, 4	+1·661
8	·193	..	5, 3	+1·620
17	·193	..	5, 3	+1·502
21	·193	..	4, 4	+1·510
25	·193	..	2, 1	+1·519
26	·193	..	5, 2	+1·476
Dec. 1	·193	..	4, 3	+1·487
4	·193	..	5, 4	+1·477
7	·193	..	3, 3	+1·445
12	·193	..	4, 3	+1·420
18	·193	..	5, 4	+1·346
25	+·193	..	3, 3	+1·299

* The zero date from which this is computed is 1928 October 26. To get temperature correction for Shortt 11, multiply that for Shortt 3 by 0·847.

It will be seen that the clock which was about a quarter of a second in advance of prediction gradually went slower, was correct before the middle of March and then fell behind, so that at the end of October

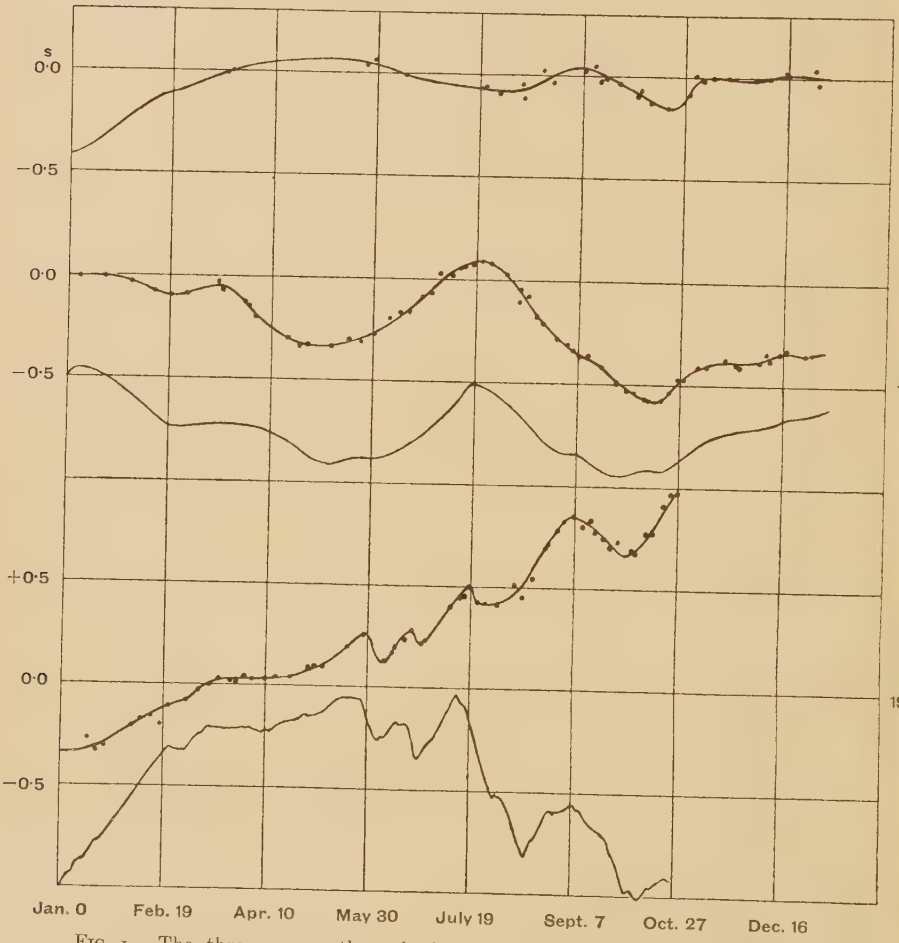


Fig. 1.—The three curves through dots show the departure of Shortt 3 from formula (3) of *M.N.*, **88**, 469, in 1927, 1928, 1929. The other two curves show the theoretical effect of the observed changes in arc in 1928 and 1929. Horizontal unit 50 days, vertical 0·5 seconds.

it was a second behind the predicted time. The actual performance of the clock for the three years 1927 1929 is shown in fig. 1. In this figure the three curves drawn through dots representing observations show the departure of the clock from prediction. Underneath the curves for 1928 and 1929 there is shown the computed effect on the error from the observed amplitude of vibration, but for 1928 the semi-

amplitude was taken as 17·55 mm. or 52′ 47″, and for 1929 17·5 mm. or 52′ 38″. It will be seen that many of the irregularities in the clock are reproduced in the effect computed from irregularities of the arc, but that after 1929 May it would be necessary to alter the standard arc by 20″ or the standard daily rate by ^{s}·016 to make the curves agree at all closely. Throughout the period 1927 March–December, from which the formula for the clock error was derived, the arc was very steady at 17·5 mm.

It should be noted here that the means for reading the arc of Shortt 3 are not sufficiently refined.* An attempt was made to read the arc to ·05 mm., and the principal change in 1928 was a decrease of 0·1 mm. about the middle of July. It will be necessary to know the arc to ·01 mm., as a variation of this amount alters the daily rate by ^{s}·00145 or the yearly rate by ^{s}·53. There should be no difficulty in doing this, and in fact the arc of No. 11 was read to this accuracy for a year after erection, but thereafter the scale was so tarnished as to be unreadable. By lacquering the scale this trouble will be removed. It might be mentioned here that there would be no serious difficulty in obtaining very accurate registration of the arc, and we believe that Dr. Loomis is doing this with his Shortt clocks. Accurate records of the arcs of two clocks, together with the difference between them, might in a comparatively short time give useful information about the remaining irregularities in these clocks. When the arcs of the Greenwich clocks were recorded it was not anticipated that the readings were so important, and in fact there was no reason to assume that they were so very steady from hour to hour as they now appear to be. Every 30 seconds, when the impulse is applied, the semi-amplitude is increased by about 6″.

It has not been thought necessary to go further into the errors of Shortt 3 for the period 1926 July to 1929 October, and to attempt to revise the formula by taking account of the observed variations of arc. The actual readings available are too rough for this to be really satisfactory. Moreover, at the beginning of this period, when it was not realised that the clocks were so accurate as they now appear to be, no serious attempt was made to control the temperature in the clock room, which fell to about 52° F. in 1926 November and rose to 64°·8 in 1929 September. In fact the agreement of the clock error with its predicted value is rather remarkable when we consider the small amount of data used for computing the temperature coefficient and the considerably higher temperature of the clock room during the last year.

There does not appear to have been any leakage in the clock case throughout the interval. In fact the readings of the two gauges indicate that the pressure actually fell. To show this the following observations have been selected at dates when equal temperatures were recorded in the clock case :—

* A scale graduated in mm. attached to the lower end of the pendulum is used.

	Temp.	Gauges. Mercury.	Oil.
	°C	mm.	mm.
1926 July 2	14·35	35·8	67·6
1927 July 26–27	14·35	35·1	60·6
1928 July 14–16	14·35	34·6	56·8
1926 July 18–19	16·0	36·15	73·0
1926 Sept. 15	16·0	36·4	71·3
1927 Aug. 29	16·0	35·3	65·6
1928 July 27–30	16·0	35·0	61·3
1928 Nov. 2	17·1	35·0	65·1
1929 Oct. 26	17·1	35·5	72·0

In view of the changes shown in 1926–1928 the change in the last year can hardly be considered as due to leakage, although it might be.

During the first run of Shortt 3, from the beginning of 1925 till 1926 July, there was a slight leak in the clock case and air was twice pumped out for the purpose of reducing the clock rate. The mean values of the pressure in inches of mercury and of the arc in millimetres are given for each month in Table II. It will be seen that as a result

TABLE II.

Rates, etc., of Shortt 3 from 1925 January to 1926 May.

1925/1926	Press.	Arc.	Daily* Rate.	Corr. for Press.	Corr. for Arc.	Corr. Rate.	Comp.	O−C.
	Inches.	mm.	s	s	s	s	s	s
Jan.	1·220	17·95	+·024	−·133	+·115	+·006	−·011	+·017
Feb.	1·276	17·95	+·067	−·167	+·115	+·015	+·024	− 9
Mar.	1·331	17·78	+·109	−·200	+·140	+·049	+·057	− 8
Apr.	1·040	18·40	+·078	−·024	+·047	+·101	+·090	+ 11
May	1·052	18·40	+·107	−·032	+·047	+·122	+·121	+ 1
June	1·077	18·35	+·133	−·047	+·055	+·141	+·151	− 10
July	0·434	20·52	+·125	+·343	−·294	+·174	+·179	− 5
Aug.	0·467	20·65	+·171	+·323	−·316	+·178	+·207	− 29
Sept.	0·478	20·55	+·215	+·316	−·299	+·232	+·233	− 1
Oct.	0·514	20·32	+·236	+·294	−·260	+·270	+·258	+ 12
Nov.	0·532	20·23	+·257	+·284	−·244	+·297	+·282	+ 15
Dec.	0·556	20·09	+·271	+·269	−·220	+·320	+·304	+ 16
Jan.	0·592	19·97	+·281	+·247	−·202	+·326	+·325	+ 1
Feb.	0·624	19·81	+·293	+·228	−·175	+·346	+·346	0
Mar.	0·671	19·71	+·310	+·199	−·159	+·350	+·364	− 14
Apr.	0·697	19·42	+·317	+·184	−·112	+·389	+·382	+ 7
May	0·750	19·28	+·332	+·152	−·090	+·394	+·399	−·005

* From *M.N.*, **88**, 473.

of leakage the arc gradually decreased, but that when air was pumped out the arc increased. It should be noticed, however, that during the second month after pumping the arc was just as large as in the first month after pumping, although the pressure had considerably increased. This would indicate that the arc took a long time to reach its maximum value after pumping, but the daily readings show that the arc rose to a nearly steady value in less than a day. The following table gives a few of the readings of pressure and arc :—

	Pressure. Inches.	Arc. mm.	Temp. °C.
1925 Mar. 30	1·35	17·75	12·6
31	1·34	17·7	12·6
Apr. 1	1·35	17·7	12·62
2	1·00	17·95	12·80
3	1·00	18·5	13·00
16	1·04	18·4	13·95
30	1·06	18·4	13·85
May 30	1·06	18·35	12·6
June 29	1·09	18·35	15·5
30	1·08	18·3	15·75
July 1	0·40	20·3	15·8
2	0·42	20·45	15·75
16	0·45	20·55	16·25
Aug. 1	0·45	20·65	16·6
17	0·48	20·65	16·45
Sept. 16	0·47	20·6	16·15

The monthly losing rates throughout this period corrected for the mean observed arc were then analysed in the form

$$a + bn + cn^2 + d(p - 1),$$

where n is the number of months from 1925 September and p the pressure in inches. The values of the important coefficients are

$$b = +^{s}\cdot0256, \qquad c = -^{s}\cdot0006, \qquad d = +^{s}\cdot606.$$

It is interesting to note that c is negative, as would be expected. The value of b deduced from our formula (3) for the error of Shortt 3 is

$$2 \times 1^{s}\cdot84 \times \cdot01 \times \cdot3 = +^{s}\cdot011,$$

and is, as might be expected, considerably less than that for the clock immediately after erection. In Table II. the rate of the clock is corrected for pressure and arc and then compared with the computed value of $a + bn + cn^2$. It will be seen that the residuals are fairly small, though by no means insignificant. They might well be associated

with the fact that the leakage never allowed the arc to get to a steady state.

The variation of the observed arc with pressure is shown in the lower part of fig. 2. It will be seen that the lower the pressure the more rapid is the increase in arc. In the upper part of the diagram there is shown the variation of the rate with pressure as deduced by allowing for the change of arc shown in the lower portion, and, in addition, $^{s}\cdot 606$

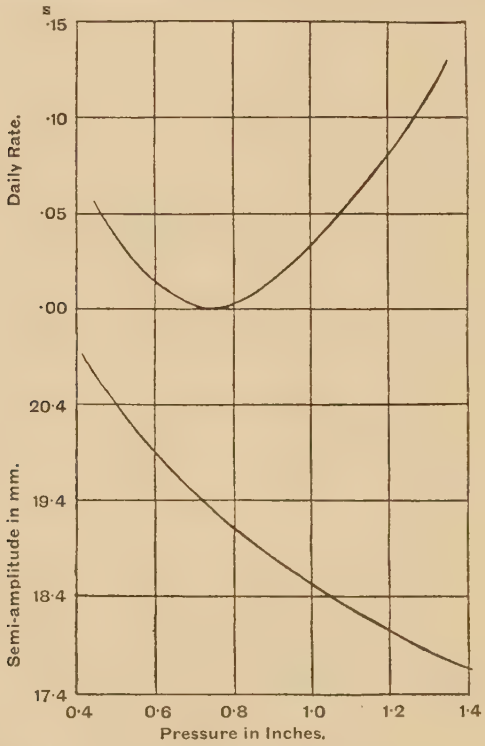


Fig. 2.—Effect of the variation of pressure on the arc and rate of Shortt 3 with the original bob.

for each inch of mercury. It will be seen that the minimum period of vibration of the pendulum is for a pressure of about 0·74 inches, and that at this pressure increase of resistance through leakage is exactly balanced by decrease of arc. This explains why so small a change of rate took place on 1925 July 1, when the pressure was reduced from 1·08 to 0·40 inches. To obtain a steady rate in the clocks it might be advisable to have the pressure about 0·7 inch, so that a slight leak would not affect the rate. This value, however, will require adjustment for the new bob. Dr. Loomis has also communicated verbally to us that he found these clocks to go much slower in a nearly perfect vacuum, presumably through a large increase in the arc.

Shortt 11.—The performance of this clock as compared with formula (6) of *M.N.*, **89**, 243, is shown in the final column of Table I., and in fig. 3. It will be seen that the clock, which was about a quarter of a second ahead of prediction at the beginning of the year, gradually lost, and throughout August and September was about 2 seconds behind; thereafter it gradually gained, and at the end of the year was only $1^{s}\cdot2$ behind. It might be pointed out that the general departure

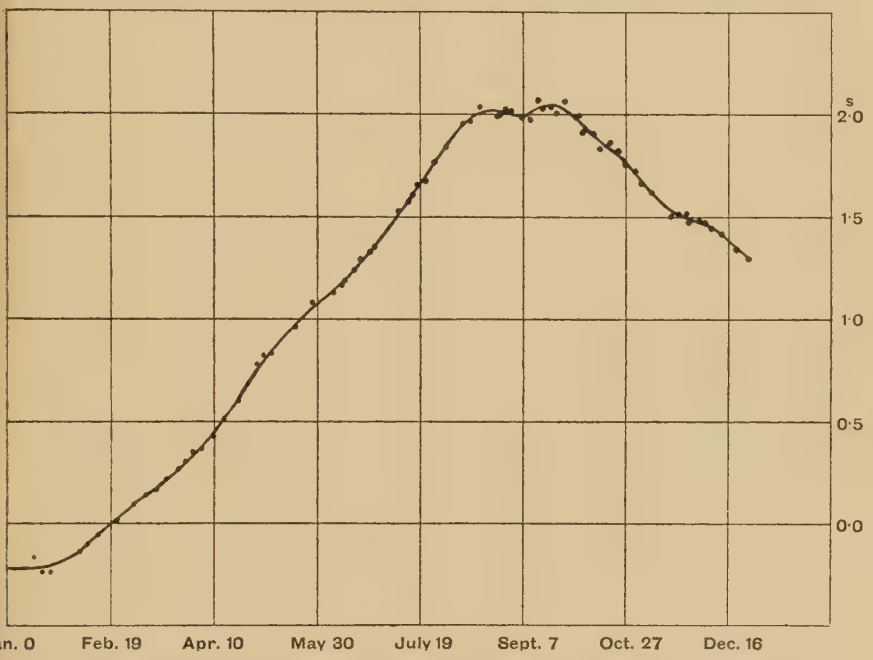


FIG. 3.—Difference between Shortt 11 and formula (6) of *M.N.*, **89**, 243, in 1929. Distance between vertical lines is 50 days, between horizontal lines 0·5 seconds.

of the clock from prediction in 1929 followed a somewhat similar course to that followed in 1928.

As there have been no arc readings of this clock since 1927 June it is impossible to go further into the cause of these irregularities, but it is worth while returning to the irregularities of this clock in the period 1926 July to 1927 June, when the arc was read with considerable accuracy. In Table III. are given the residuals of No. 11 as given in *M.N.*, **89**, 244, together with the correction to reduce them to an arc of 55′ 20″·1, a value which was selected so as to make the errors on 1927 March 19 and 1927 May 26 equal to zero. This is quite a reasonable value of the arc, which in 1926 August was about 54′ 45″, and slowly increased to 55′ 12″ by 1927 May, although at the end of October the readings were less than 54′ 30″. It will be seen that the smallness of the arc for the first year after this clock was erected explains the larger

part of the irregularities of the clock, but unfortunately the lack of knowledge of later values of the arc makes it impossible for us to enter further into this question.

TABLE III.

Effect of Variations of Arc on Shortt 11 *from* 1926 *August to* 1927 *June.*

1926/1927.	Error of No. 11.	Corr. for Arc.	Final Residual.	Arc.
	s	s	s	′ ″
Aug. 8	+7·94	−5·04	+2·90	
				54 46
18	7·42	−4·75	+2·67	
				43
28	6·87	−4·44	+2·43	
				47
Sept. 7	6·31	−4·16	+2·15	
				47
17	5·87	−3·89	+1·98	
				40
27	5·26	−3·55	+1·71	
				47
Oct. 7	4·72	−3·31	+1·41	
				52
17	4·19	−3·08	+1·11	
				48
27	3·70	−2·81	+0·89	
				32
Nov. 6	3·09	−2·43	+0·66	
				33
16	2·56	−2·04	+0·52	
				40
26	2·18	−1·71	+0·47	
				46
Dec. 6	1·91	−1·40	+0·51	
				55
16	1·69	−1·19	+0·50	
				56
26	1·41	−0·99	+0·42	
				54 58
Jan. 5	1·13	−0·80	+0·33	
				55 3
15	0·96	−0·67	+0·29	
				2
25	0·77	−0·51	+0·26	
				12
Feb. 4	0·69	−0·45	+0·24	
				10
14	0·52	−0·36	+0·16	
				10
24	0·36	−0·28	+0·08	
				9
Mar. 6	0·26	−0·19	+0·07	
				11
19	+0·08	−0·08	0·00	
				12
May 26	−0·35	+0·35	0·00	
				13
June 12	−0·35	+0·46	+0·11	

The Effect of Boundary Conditions on the Equilibrium of a Star.

By A. S. Eddington, F.R.S.

The problem examined in this paper may be stated as follows: Suppose that we are given the mass M and the radius R of a star, the opacity of its material as a function of temperature and density, the molecular weight (also as a function of temperature and density), and the proportionate distribution of the source of energy in different parts of the star, then it is possible to calculate rigorously the rate of radiation L. Now take another star with the same specification, except that the outer layers down to a depth a little below the photosphere are replaced by material of ten times greater intrinsic opacity. What will be the resulting change of L?

The answer is that the change is entirely insignificant. The photospheric layers have no special part in determining the luminosity and only affect the problem in so far as the small amount of mass involved contributes to the average properties of the star. This was, I believe, a novel point when I first developed it in my theory of stellar equilibrium in 1916; but it seems to have been unchallenged until E. A. Milne opposed it last November.* I have never given a formal proof, as it seemed sufficiently obvious when the physical conception of a stream of radiation flowing outwards through the whole interior was grasped; but I have often verified it in my own numerical calculations, and in one case I have published definite figures. These relate to a star with a point-source of energy at the centre.† Two solutions were calculated for stars with the same L, but the mass of one up to a radius $8{\cdot}1 \cdot 10^{11}$ cm. was $9{\cdot}687 \cdot 10^{33}$ gms., and of the other $9{\cdot}706 \cdot 10^{33}$ gms. At this radius the pressure was about $\frac{1}{300}$ of the central pressure, so that the exterior mass to be supported must be small (about $0{\cdot}26 \cdot 10^{33}$ gms.). Continuing the solutions outwards, in one of the stars p_G comes down to zero before p_R, so that the star is finished off (*i.e.* p_R brought down to the necessary boundary value) by a layer of matter wholly supported by radiation-pressure; this means that the outer layers have an opacity as high as is possible without explosion. In the second star p_R comes down to its surface value whilst p_G is still comparatively large, so that the star is finished off (p_G brought down to zero) by a layer of perfectly transparent material. This, then, is a direct calculation of the effect on the relation of L to M of making a large change in the boundary conditions, viz. substituting perfectly transparent material for material as opaque as possible to a depth far greater than the ordinary photospheric depth. The effect of a relatively small change such as that contemplated in the first paragraph will, of course, be proportionately smaller.

Professor Milne claims that "the surface conditions have a completely dominant effect." "In any cooling problem boundary conditions are paramount." I gather that he considers my calculation of stellar luminosity from the mass, radius, and physical properties of the

* *M.N.*, **90**, 17, 1929. See, for example, the last 7 lines on p. 19.

† *Internal Constitution of the Stars*, p. 124.

interior to be vitiated by insufficient attention to the boundary conditions; and he believes that the photospheric opacity (as one of the boundary conditions) is a leading factor in determining the luminosity. That is the challenge which I here take up.

I think Milne will agree with me that this is a purely mathematical question, and no question of judgment as to plausibility of physical hypotheses is involved. In the last resort the acceptance or rejection of a hypothesis is a matter of opinion; but there is no excuse for any difference of opinion as to the consequences deducible from given premises; one or other of the contestants must have blundered. It seems unfortunate that the investigation of the interior of a star should have been the field not only for legitimate differences of opinion, but for disagreement as to the correctness of deduction; it is very much as though in mathematics there were several rival theories as to what is the correct solution of a quadratic equation. I think that it would have been reasonable to hope that before adding further to this confusion Professor Milne would have made a more definite attempt to justify the statements above quoted.

Milne makes it clear that the supposed dependence of luminosity on the photospheric opacity is put forward as a general condition, not dependent on any particular model. As mentioned above, I have already treated the point-source model, and according to my calculations Milne is wrong. I shall here take a model (probably nearer to stellar conditions) which can be treated analytically so that the reader may verify my result without undue labour. Apart from the particular occasion which has elicited this investigation, it has some interest as showing how fast a disturbance of surface conditions dies out as we go down into the interior. It may be added that the case chosen is one of completely neutral equilibrium, in which, according to Milne, the sensitivity to boundary conditions is especially great.

Although the actual figures relate to a particular model, it is clear from the formulæ obtained that my result is independent of the specification of the interior part of the star hotter than, say, 500,000°, since the effect of the surface disturbance is shown to have died out before this part of the star is reached.

The model star to which I apply the test proposed in the opening paragraph is specified as follows: * The source of energy is evenly distributed through the mass (η = const.); the opacity of the material varies according to the law $k \propto \rho/\mu T^3$. This law may be written $k = \kappa p_G/p_R$, where κ represents the intrinsic opacity of the material. The intrinsic opacity κ_1 of the photospheric layers is not necessarily the same as κ for the interior.

Since, for η = const., H/g is equal to the constant $L/4\pi GM$, the usual equation

$$dp_R = (kH/cg)dP$$

gives

$$dp_R = \sigma y dP \quad . \quad . \quad . \quad . \quad (1)$$

* I use the notation and formulæ of *Internal Constitution of the Stars*, ch. vi.

where $$y = p_G/p_R \qquad \sigma = \kappa L/4\pi cGM \quad . \quad . \quad . \quad (2)$$

Let

$$z = p_R/P = 1/(1+y) \quad . \quad . \quad . \quad (3)$$

Then

$$\frac{dz}{dP} = \frac{1}{P}\left(\frac{dp_R}{dP} - \frac{p_R}{P}\right) = \frac{1}{P}(\sigma y - z) = \frac{1}{P}\left(\sigma\left(\frac{1}{z} - 1\right) - z\right)$$

so that

$$\frac{zdz}{z^2 + \sigma z - \sigma} = -\frac{dP}{P} \quad . \quad . \quad . \quad (4)$$

Let

$$z^2 + \sigma z - \sigma = (z - \alpha)(z + \alpha') \quad . \quad . \quad . \quad (5)$$

Then the solution of (4) is

$$\frac{\alpha}{\alpha + \alpha'} \log (z - \alpha) + \frac{\alpha'}{\alpha + \alpha'} \log (z + \alpha') = -\log P + \text{const.} \quad (6)$$

When $\kappa_1 = \kappa$ the well-known solution (applying to the whole star) is $z = $ const., viz. the $1 - \beta$ of ordinary theory; hence, by (4) $z^2 + \sigma z - \sigma = 0$, so that $z = \alpha = 1 - \beta$.

When $\kappa_1 \neq \kappa$, write $z = \alpha + \epsilon$, so that ϵ represents the disturbance of z due to the modified boundary condition. Then (6) becomes

$$1/P = \text{const.} \times \epsilon^{\alpha/(\alpha+\alpha')}(\alpha + \alpha' + \epsilon)^{\alpha'/(\alpha+\alpha')} \quad . \quad . \quad (7)$$

We see from (7) that when ϵ is large compared with $\alpha + \alpha'$, $\epsilon \propto 1/P$; when ϵ is small compared with $\alpha + \alpha'$, $\epsilon \propto 1/P^{1+\alpha'/\alpha}$. Since by (5) $\alpha' > \alpha$, the latter variation is $\epsilon \propto 1/P^{2+}$. For stars of not unduly large mass $1 + \alpha'/\alpha$ is not much greater than 2, so that practically the falling-off of ϵ as we go inwards is between the limits $1/P$ and $1/P^2$.

Take a star which originally (*i.e.* for $\kappa_1 = \kappa$) has the mass, brightness, and radius of the sun. Then $\alpha = \cdot 05$ (the value of $1 - \beta$ for the ordinary model). By (5)

$$\frac{1}{\alpha} - \frac{1}{\alpha'} = 1.$$

Hence $$\alpha' = 1/19; \quad \text{and} \quad \alpha + \alpha' = \cdot 103.$$

I shall take the outer region to which κ_1 is intended to apply to extend to a depth where the pressure has a prescribed value P_1, corresponding roughly to a temperature of 30,000° (say $2 \cdot 10^4$ dynes). Variation of boundary conditions will mean variation of the physical characteristics (more especially the intrinsic opacity) of the material above this level. The outer layers can adjust themselves to correspond to any prescribed values of H and g or equivalently to any prescribed L, M, R; and we can, of course, always find a depth at which the total pressure P has the above chosen value; but the temperature at this level or, what comes to the same thing, the ratio $p_R/P = z$ will depend on the intrinsic opacity of the material above this level. Thus the value z_1 at the fixed pressure P_1 gives the boundary condition which the solution of the equations for the interior must satisfy.

To test the effect of a change of boundary conditions, we shall compare solutions corresponding to $z_1 = \cdot 05, \cdot 55$. Since $\alpha = \cdot 05$, these give $\epsilon_1 = 0, 0{\cdot}5$. The latter represents a very large increase of κ_1, probably much more than the tenfold increase suggested in our opening paragraph.

Considering the disturbed case $\epsilon_1 = 0{\cdot}5$, we trace the course of ϵ as we go downwards. At first it is larger than $\alpha + \alpha'$, so that ϵ varies not much more rapidly than $1/P$. Below the level $10\ P_1$ it is smaller and diminishes more nearly as $1/P^2$. It will have come down to $\cdot 001$ at a level about $100\ P_1$ (about 2 atmospheres pressure) where the temperature is about 100,000°. Here we draw a second boundary (P_0), so that $\epsilon_0 = \cdot 001$.

The region between 100,000° and 30,000° is the region of recovery from the disturbance caused by our tampering with the material outside the 30,000° level. Within our second boundary P_0, ϵ is a small quantity whose square can be neglected, so that the discussion is simplified. It will be noticed that ϵ fades out very rapidly below P_0; *e.g.* at 1,000,000° $\epsilon = 10^{-11}$, and we have still not penetrated through any appreciable fraction of the mass.

We treat first the part of the star interior to P_0. We make a comparison with a star for which $\epsilon = 0$, having the same central pressure P_c and the same α (*i.e.* the same $\kappa L/M$ by (2) and (5)). We find the mass and radius at the end of the investigation, and since α is known we shall thus determine κL corresponding to this mass and radius and the boundary condition $\epsilon_1 = 0{\cdot}5$.

The comparison is made at the same distance from the centre ($\delta r = 0$). The change δg is of much smaller order of magnitude than δP and $\delta\rho$ and will be neglected. Since $dP = -g\rho dr$ we have

$$\frac{d\delta P}{dP} = \frac{\delta\rho}{\rho} \quad . \quad . \quad . \quad . \quad (8)$$

Also since $$p_G \propto \rho T \propto \rho p_R^{\frac{1}{4}},$$

$$\frac{\delta\rho}{\rho} = \frac{\delta p_G}{p_G} - \frac{1}{4}\frac{\delta p_R}{p_R} \quad . \quad . \quad . \quad . \quad (9)$$

And since

$$p_R = (\alpha + \epsilon)P \qquad p_G = (1 - \alpha - \epsilon)P \quad . \quad . \quad (10)$$

(9) reduces to

$$\frac{\delta\rho}{\rho} = \frac{3}{4}\frac{\delta P}{P} - \eta \qquad \eta = \epsilon\left(\frac{1}{1-\alpha} + \frac{1}{4\alpha}\right) \quad . \quad . \quad (11)$$

Eliminating ρ between (8) and (11)

$$\frac{d\delta P}{dP} - \frac{3}{4}\frac{\delta P}{P} + \eta = 0 \quad . \quad . \quad . \quad (12)$$

which gives on integration, since $\eta \propto \epsilon \propto P^{-2}$,

$$\frac{\delta P}{P} = \frac{4}{7}\eta \qquad \frac{\delta\rho}{\rho} = -\frac{4}{7}\eta \quad . \quad . \quad . \quad (13)$$

At the boundary we find from (11), $\frac{4}{7}\eta_0 = 3{\cdot}5\,\epsilon_0 = {\cdot}0035$.

The pressure at r_0 being (in the disturbed star) $P_0 + \delta P_0 = 1{\cdot}0035\,P_0$, we have to shove out the boundary to $r_0 + \delta r_0$, so as to reach the standard value P_0. Calculating the pressure gradient at P_0 for the sun, we find $\delta r_0 = 50$ km. approximately.

The change in the total mass (as far as the level P_0) can also be calculated. Since this is exceedingly small, it will be sufficient to find an upper limit. The most convenient formula is

$$-\frac{dM_r}{dP} = \frac{4\pi r^2 \rho dr}{g\rho dr} = \frac{4\pi r^4}{GM_r}.$$

Hence

$$-d(\delta M) = \frac{4\pi\delta(r^4)}{GM_r}\,.\,dP,$$

where δr denotes the change of r for the same value of P. We have $\delta r = 50$ km. at P_0 and it decreases approximately as $P^{-\frac{1}{5}}$. Using a maximum value of r so as to obtain an upper limit, we easily calculate that $\delta M/M < 10^{-12}$.

Thus the effect of the alteration of the opacity of the photospheric region is to cause an expansion of about 50 kilometres in the radius as far as P_0, and an alteration of less than 1 part in 10^{12} in the mass.

Turning now to the part of the star between P_0 and P_1, the expansion will be more pronounced; in the undisturbed model this region occupies roughly $\frac{1}{15}$ of the stellar radius. It is difficult to trace the change $\delta\rho$ between P_0 and P_1, but at P_1 it is found from the prescribed value of z_1. Since p_R is changed from ·05 P_1 to ·55 P_1, T_1 is increased in the ratio $11\frac{1}{2}$; and since p_G is changed from ·95 P_1 to ·45 P_1, ρ is changed in the ratio $\frac{9}{19} \div 11\frac{1}{2} = 0{\cdot}26$. If the density were reduced in this ratio throughout the region the width would be increased in the reciprocal ratio—that is to say, nearly fourfold.* We may perhaps expect the actual increase to be half this upper limit, or 8 per cent. of the radius of the star.

In the undisturbed model the mass outside P_0 is easily calculated; it is about $3{\cdot}10^{-9}$ of the whole mass. This will be increased a little in the disturbed model owing to the diminution of g through the expansion; but the effect is obviously trifling.

Summary.

To show that the luminosity of a star is practically independent of the physical properties of the material forming its surface layers, I have compared the solutions for two stars with the same internal material, but with very different material near the surface. The change actually considered is to blanket the star with very opaque surface material so that at a depth where the temperature is about 30,000° radiation-pressure is 55 per cent. of the whole pressure instead of

* Since $dP = -g\rho dr$. We neglect the change of g which would slightly increase the width.

5 per cent. for the sun. For convenience the stars compared have the same L/M and therefore slightly different masses; but this could readily be converted into a comparison of the L's for the same mass. It is shown that the above change of surface conditions leads to a change of mass less than 1 in 10^{10} (L/M being the same).

Another consequence of the blanketing is an expansion of the radius of the star by an amount estimated roughly at 8 per cent. This expansion occurs almost entirely in the region with temperature below 100,000°, and it is shown that the whole disturbance dies out quickly in the interior. I have already called attention to the fact that the actual radius of a star is likely to differ somewhat from the radius obtained by extrapolating the solution for the interior (*Int. Const. of the Stars*, § 67), and the change here found is well below the limit there contemplated. With the generally accepted formula for the opacity, the luminosity is very insensitive to changes of radius. The effect on L is therefore of no practical importance, though it is, of course, larger than that corresponding to the insignificant change of mass found above.

On Professor Milne's Treatment of the Mass-Luminosity Problem.
By A. S. Eddington, F.R.S.

Professor Milne's recent paper * contains criticisms of my theory of the luminosities of stars to which I reply in another paper; the present paper relates to the development which he proposes to substitute. I do not think he has realised how disconnected is his field of investigation from the problems of the interior.

1. The first stage of his discussion culminates in § 15 in his main equation (24), which is supposed to be derived from the equation of mechanical equilibrium together with the assumption that the *outer layers* are gaseous. The meaning of this equation becomes clearer if for θ, ϕ, x we substitute the more recognisable quantities P, ρ, etc. according to the definitions given by Milne. We can also substitute $\frac{1}{3}aP_1^3/\rho_1^4$ for the left side in accordance with (17). Then it is found that (24) reduces to

$$\frac{d\rho}{dr} = -\frac{3}{4}\frac{g\rho^2}{P} \quad . \quad . \quad . \quad . \quad (1)$$

In conjunction with the hydrostatic equation $dP = -g\rho dr$, this gives

$$\frac{d\rho}{dP} = \frac{3}{4}\frac{\rho}{P} \quad . \quad . \quad . \quad . \quad (2)$$

so that

$$P \propto \rho^{\frac{4}{3}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The above equations refer to the photospheric region.

* *M.N.*, **90**, 17, 1929.

So far as we know (3) is untrue. But right or wrong, it is obvious that neither it nor its equivalent (1) can have been deduced almost "out of nothing" in the way Milne would have us suppose. On examining his work we find he has assumed it. "We shall impose the restriction that $\phi(\theta)$ tends to a definite limit ϕ_1 as $\theta \to 0$," though we are told that the restriction is "only a mathematical device." This oversight seems to vitiate the paper so far as its bearing on the stellar interior is concerned.

The value of $d\rho/dr$ in the photosphere can be found by methods developed principally by Professor Milne himself. It is, of course, not generally equal to the value given by (1), so that in employing (24) Milne is ignoring the actual boundary conditions of a star. He has remarked in another connection that "a small change of boundary conditions may have a devastating effect."

It is important to realise that Milne's (24) is not a deduction from his theory of the interior together with an assumption. It is the assumption alone; that is to say, working backwards from the assumption (3) together with the gas equation for the outer layers, we obtain his equation (24) without any reference to the interior. It appears to me that in the constructive part of Milne's paper all his references to the interior of the star should be deleted, since he has no mathematical linkage between the interior and the quantities he evaluates and he has therefore no foundation even for conjecture.

2. From § 15 onwards he is largely occupied with a certain quantity C. Omitting absolute constants, C denotes the photospheric value of P^3/ρ^4M^2. By stating it in plain terms some of the mysticism surrounding it may be dispelled. The constant C "is a telescoped version of the structure of the star between centre and boundary" (§ 28). This only means that like every other combination of photospheric data it provides a boundary condition for the differential equations of the interior.

It is well known that when T_1 and g_1 are given together with the physical properties of the material (intrinsic opacity, molecular weight, ionisation potentials) the condition of the outer layers is completely determined, and the photospheric values P_1, ρ_1 can be calculated by methods developed chiefly by Milne. Thus C is a function of T_1, g_1, M or equivalently of T_1, L, M. This is the four-term relation so often referred to in the paper. But any other photospheric characteristic is a function of T_1, L, M, and could replace C in the four-term relation.

For completely homologous stars P^3/ρ^4M^2 is the same at corresponding points. Hence Milne concludes that homologous stars have the same C. But the photospheres of two homologous stars are not "corresponding points," and the conclusion is in general incorrect. Here his assumption $P \propto \rho^{4/3}$ comes in; it makes P^3/ρ^4M^2 constant in the outer part of the star so that the question of correspondence of the points does not arise. Without this arbitrary assumption there is no connection between homology and C.

Of the two reasons for selecting C rather than any other combination of data for consideration, the first (that it enters into a four-term relation)

is common to all photospheric quantities, the second (that it is constant for stars of homologous density) is untrue in general.

3. Professor Milne advocates a new method of studying the structure of a star, viz. to find the relations between the observables of a star which are independent of arbitrary assumptions about the unknown interior. To find relations between observables, *e.g.* a mass-luminosity relation, is the aim of older methods; everything turns therefore on what is meant by arbitrary assumptions. I do not consider it arbitrary to assume that the star consists of chemical elements not too unlike those familiar to us, that there are no sinks or places where energy is continually disappearing, that it is governed by the laws of physics which have come to be generally accepted. These assumptions may be doubtful, but they are not arbitrary or speculative. Professor Milne rejects these and in exchange for all the information obtained by other methods "we are led to infer a single definite fact about the internal density distribution." This fact is the value of C, which is suspected to be approximately constant.

To call C a fact about the internal density is a sophistry, apparently occasioned by Milne's philosophical doctrine that the only thing we can really know about the inside is the outside. C is a fact about the photosphere. The suggestion occurring frequently in Milne's paper that stars with the same C have some similarity of internal distribution of density is unfounded speculation. It is the drawback of the new method that such conjectures can be made irresponsibly, because we are not allowed to apply the accepted laws of physics to the interior to examine whether the conjecture is supported or not by the present state of knowledge. Actually, using all our resources, it is scarcely possible to decide how the density distribution differs between massive stars and light stars. Milne's only resources are a photospheric constant (which I think I have elsewhere shown to be irrelevant) together with the equation of mechanical equilibrium (which, as shown in § 1, he does not actually use). I cannot see that he is in a position to make any suggestion on the subject.

To sum up—Professor Milne's investigation is solely concerned with the photosphere. However useful such an investigation may be, it naturally will not yield the results that can be obtained by a study of the interior. He calls attention to the absence of assumptions about the interior. "For all our analysis is concerned the stars may be liquid in their interiors" (§ 10). My own analysis of the interior of a star is equally free from assumptions—about the photosphere. Milne is by no means averse from employing our physical knowledge in the photosphere; and in fact C itself is an "unobservable." Without disparaging his valuable theory of the outer layers, I think it would be optimistic to regard C as ranking in accuracy and trustworthiness with comparatively well-determined "unobservables" of the interior such as the mean temperature. However, it is beside the point to discuss whether the theory of the interior or of the exterior is the more secure. My criticism of Milne's paper is, that without justification it claims to be a mode of finding out about the interior.

Remarks on the Schwarzschild-Milne Model of the Outer Layers of a Star. By Eberhard Hopf.

(*Communicated and translated by Prof. E. A. Milne.*)

The following note contains a few mathematical remarks on the Schwarzschild-Milne model of a stellar atmosphere of infinite optical depth, consisting of plane-parallel layers in radiative equilibrium. As is well known, the principal problem is the determination of the mean radiation-intensity $B(\tau) \propto T^4$ as a function of the optical depth τ measured from the surface. In § 1 it is shown, in continuation of earlier mathematical notes, that the exact solution of Milne's problem is of the form

$$B(\tau) = \tfrac{3}{4}F(\tau + \phi(\tau)), \quad \tfrac{1}{2} < \phi(\tau) < 1,$$

where πF denotes the net flux. The function $\phi(\tau)$ is thereby determined as the solution of a non-homogeneous integral equation which can be solved by successive approximation. § 2 contains a few mathematical remarks on the mechanism of the method of solution of the problem by successive approximation. It is perhaps not without theoretical interest that in § 3 we deduce the exact value of T_e/T_0, where T_e is the effective temperature and T_0 is the boundary temperature. We find, in fact,

$$T_e^4 = \frac{4}{\sqrt{3}} T_0^4,$$

a result which offers a certain improvement on the otherwise satisfactory accuracy of Milne's approximation.

§ 1. *Rigorous Solution of the Problem.*

The Schwarzschild-Milne model of a stellar atmosphere is that of a medium stratified in parallel planes, of infinite optical depth ($0 \leqslant \tau < \infty$); $I(\tau, \theta)$ denotes the total intensity of radiation at optical depth τ in a direction making an angle θ with the outward normal to the planes of stratification. As is well known, the fundamental equations of the radiation field for a steady state are

$$\cos\theta \frac{dI(\tau,\theta)}{d\tau} = I(\tau,\theta) - B(\tau), \quad . \quad . \quad . \quad (1)$$

$$B(\tau) = \tfrac{1}{2}\int_0^\pi I(\tau,\theta) \sin\theta d\theta \quad . \quad . \quad . \quad (2)$$

From (1) and (2) we deduce in the usual way the constancy of the net flux πF, in the form

$$F = 2\int_0^\pi I(\tau,\theta) \cos\theta \sin\theta d\theta, \quad F = \text{constant} \quad . \quad (3)$$

The problem consists in the determination of the function $B(\tau)$ for given F, with the boundary conditions

$$I(o, \theta) = o \quad (\tfrac{1}{2}\pi < \theta \leqslant \pi), \qquad \lim_{\tau \to \infty} \frac{I(\tau, \theta)}{B(\tau)} = 1 \quad (o \leqslant \theta \leqslant \tfrac{1}{2}\pi) \quad (4)$$

for the inward and outward flowing beams. The significance of this problem in the theory of stellar atmospheres arises of course from its relation with the temperature-distribution in the outer layers, which is given by

$$T_\tau^4 = \frac{B(\tau)}{F} T_e^4 \quad . \quad . \quad . \quad . \quad (5)$$

where T_τ is the temperature at depth τ and T_e is the effective temperature of the radiation emerging at the layer $\tau = o$.

Following Milne's * procedure, we can express $I(\tau, \theta)$ in terms of $B(\tau)$, by means of equation (1) and conditions (4), and then by introducing for $I(\tau, \theta)$ in (2) we obtain the homogeneous integral equation

$$B(\tau) = \tfrac{1}{2}\int_0^\infty B(t) E i(|\tau - t|) dt \quad . \quad . \quad . \quad (6)$$

which the function $B(\tau)$ must satisfy ; the functions

$$E i_n(x) = \int_1^\infty \frac{e^{-sx}}{s^n} ds, \quad E i(x) = E i_1(x)$$

are well known. In (6) the dependence on the given constant F is not explicit. As Milne † observes, we can easily put this dependence in evidence by inserting the expressions for $I(\tau, \theta)$ in terms of $B(\tau)$ directly in equation (3) instead of in (2). We then easily derive the integral equation

$$\tfrac{1}{2}F = \int_\tau^\infty B(t) E i_2(t - \tau) dt - \int_0^\tau B(t) E i_2(\tau - t) dt \quad . \quad (7)$$

for the determination of $B(\tau)$. Equation (7) can accordingly be considered the fundamental equation of the Schwarzschild-Milne problem.

In the following paragraph we provide a direct path for the determination of the explicit connection between $B(\tau)$ and the flux-constant F. It can be shown rigorously ‡ that Milne's integral equation (6) possesses solutions of the form

$$B(\tau) = c(\tau + \phi(\tau)) \quad . \quad . \quad . \quad (8)$$

where c is an arbitrary constant and $\phi(\tau)$ is a determinate function lying between the limits

$$\tfrac{1}{2} < \phi(\tau) < 1 \quad . \quad . \quad . \quad . \quad (8a)$$

It can be shown further § that for $c > o$, solutions (8) are the only positive

* *M.N.R.A.S.*, **81**, 361, 1921.
† *Phil. Trans.*, A, **228**, 421, 1929, in particular p. 425.
‡ *Zeit. für Phys.*, **46**, 374, 1928.
§ *Ibid.*, **48**, 155, 1928.

solutions of (6). In order to have the solution to the problem it simply remains, therefore, to express c in terms of F. We find

$$c = \tfrac{3}{4}\mathrm{F},$$

as can be seen without difficulty as follows. Following Eddington,* multiply (1) by $\cos\theta \sin\theta d\theta$ and integrate. Taking account of (3), we obtain

$$\int_0^\pi \mathrm{I}(\tau,\theta)\cos^2\theta \sin\theta d\theta = \tfrac{1}{2}\mathrm{F}\tau + \text{constant} \quad . \quad . \quad (9)$$

Now in virtue of (8) and (8a), $\mathrm{B}(\tau)/\tau \to c$ as $\tau \to \infty$, and thus in virtue of the second of conditions (4), which is easily seen to be satisfied also by the inward moving radiation (*i.e.* for $\frac{1}{2}\pi \leqslant \theta \leqslant \pi$), we have

$$\frac{\mathrm{I}(\tau,\theta)}{\tau} \to c \quad \text{as} \quad \tau \to \infty .$$

Hence, dividing (9) by τ and proceeding to the limit we have

$$\tfrac{1}{2}\mathrm{F} = c\int_0^\pi \cos^2\theta \sin\theta d\theta = \tfrac{2}{3}c,$$

i.e. $c = \frac{3}{4}\mathrm{F}$. Hence by (8) the solution is

$$\mathrm{B}(\tau) = \tfrac{3}{4}\mathrm{F}(\tau + \phi(\tau)) \quad . \quad . \quad . \quad (10)$$

With regard to the function $\phi(\tau)$, we find by inserting (10) in (6), after a few transformations, that $\phi(\tau)$ satisfies the non-homogeneous integral equation

$$\phi(\tau) = \tfrac{1}{2}\int_0^\infty \phi(\tau)\mathrm{E}i(|\tau - t|)dt + \tfrac{1}{2}\mathrm{E}i_3(\tau) \quad . \quad . \quad (11)$$

From certain theorems previously established † it follows that equation (11) possesses precisely one bounded solution, $\phi(\tau)$; the solution lies, moreover, between the limits (8a). It is thus proved that the Schwarzschild-Milne problem possesses precisely one solution. The law of temperature distribution is

$$\mathrm{T}_\tau^4 = \tfrac{3}{4}(\tau + \phi(\tau))\mathrm{T}_e^4 \quad . \quad . \quad . \quad (12)$$

The formula for the intensity-distribution at the surface takes the form

$$\mathrm{I}(0,\theta) = \tfrac{3}{4}\mathrm{F}(\cos\theta + \int_0^\infty e^{-s}\phi(s\cos\theta)ds) \qquad (0 \leqslant \theta < \tfrac{1}{2}\pi).$$

§ 2. *On the Approximate Solution of the Problem.*

The discussion of the integral equations (6) and (11) is carried out most conveniently by introducing the symbol

$$\tfrac{1}{2}\int_0^\infty f(t)\mathrm{E}i(|\tau - t|)dt = \Lambda(f).$$

* *Internal Constitution of the Stars*, § 225, 1926.
† *Zeit. für Phys.*, **46**, 374, 1928.

Similarly we make use of the abbreviation

$$(f, g) = (g, f) = \int_0^\infty f(t)g(t)dt.$$

For example, we have

$$\Lambda(1) = 1 - \tfrac{1}{2}\mathrm{E}i_2(\tau) \quad . \quad . \quad . \quad . \quad (13)$$

If the operations consisting of repeated applications of the operator Λ are denoted by $\Lambda_0(f) = f, \Lambda_1 = \Lambda, \Lambda_2 = \Lambda(\Lambda_1), \ldots \Lambda, {}_{n+1} = \Lambda(\Lambda_n), \ldots,$ the theorem holds that $\Lambda_n(1) \to 0$ as $n \to \infty$.* Hence, in virtue of (13)

$$\tfrac{1}{2}(\mathrm{E}i_2 + \Lambda(\mathrm{E}i_2) + \Lambda_2(\mathrm{E}i_2) + \ldots + \Lambda_n(\mathrm{E}i_2) + \ldots) = 1 \quad (13a)$$

The integral equation (11) may now be written

$$\phi = \Lambda(\phi) + \tfrac{1}{2}\mathrm{E}i_3 \quad . \quad . \quad . \quad . \quad (14)$$

At the point $\tau = 0$ this gives

$$\phi(0) = \tfrac{1}{2}(\phi, \mathrm{E}i) + \tfrac{1}{4} \quad . \quad . \quad . \quad . \quad (15)$$

Before we discuss the solution of (14), the following remark may be made. The kernel $\mathrm{E}i(|\tau - t|)$ of the integral operator Λ is symmetrical. The symmetry-property can by means of the above symbols be expressed also in the formula

$$(f, \Lambda(g)) = (g, \Lambda(f)) \quad . \quad . \quad . \quad . \quad (16)$$

for both sides of (16) are simply

$$\tfrac{1}{2}\int_0^\infty \int_0^\infty f(t)g(t)\mathrm{E}i(|\tau - t|)d\tau dt.$$

The rigorous solution of (14) is given by the (convergent) Neumann series,

$$\phi = \tfrac{1}{2}[\mathrm{E}i_3 + \Lambda(\mathrm{E}i_3) + \Lambda_2(\mathrm{E}i_3) + \ldots + \Lambda_n(\mathrm{E}i_3) + \ldots] \quad (17)$$

Although by this formula an exact solution of the Schwarzschild-Milne problem is achieved, for the practical calculation of $\phi(\tau)$ series (17) is quite useless owing to the slowness of the convergence. Further, the functions $\Lambda(\mathrm{E}i_3), \Lambda_2(\mathrm{E}i_3) \ldots$ are of very complicated form. If in the following we go into the question of a rapidly converging approximation-procedure for the solution of (14), this must not be considered as an attempt to improve the existing known approximations (which would be a piece of purposeless labour), but rather to throw light on the mathematical mechanism of the earlier methods of approximation.

The approximations of Jeans, Milne, and Eddington are obtained by correcting the partial sums of the poorly converging series (17) by the addition of definite terms determined by a procedure which can be continued indefinitely. The general rule for the nth approximation in the sense of Jeans and Milne is

$$\begin{aligned} \phi_1 &= 1 \\ \phi_2 &= c_2\Lambda(1) + \tfrac{1}{2}\mathrm{E}i_3 \\ &\cdots \\ \phi_n &= c_n\Lambda_{n-1}(1) + \tfrac{1}{2}[\mathrm{E}i_3 + \Lambda(\mathrm{E}i_3) + \ldots + \Lambda_{n-2}(\mathrm{E}i_3)] \quad . \quad (18) \end{aligned}$$

* *Zeit. für Phys.*, **46**, 374, 1928.

where the c_n's are an appropriately chosen sequence of constants. Now relation (15) holds rigorously. If we determine c_n so that the nth approximation ϕ_n satisfies (15) exactly, we find after a few simple calculations

$$c_n = \frac{\Lambda_{n-1}(\mathrm{E}i_3)_{\tau=0}}{\Lambda_{n-1}(\mathrm{E}i_2)_{\tau=0}} \quad \text{(Schwarzschild, Jeans).}$$

These approximations are thus so constructed that each one satisfies the condition of radiative equilibrium at the boundary. They converge appreciably better than the Neumann series (17). According to Milne's procedure, one obtains still better approximations if one chooses as the controlling condition not (15) but the equally rigorous relation

$$(\phi, \mathrm{E}i_2) = \tfrac{1}{3} \quad . \quad . \quad . \quad . \quad (19)$$

which is easily deduced from (10) and (7) ($\tau = 0$). Milne's approximations thus satisfy the control-condition (19).* For the c_n's we obtain easily the formula

$$c_n = \frac{(1, \Lambda_{n-1}(\mathrm{E}i_3))}{(1, \Lambda_{n-1}(\mathrm{E}i_2))} \quad \text{(Milne).}$$

The first approximation,

$$\phi_1 = c_1 = \tfrac{2}{3}, \quad \mathrm{B}_1(\tau) = \tfrac{3}{4}\mathrm{F}(\tau + \tfrac{2}{3}),$$

has already proved itself useful in the theory of stellar atmospheres. The second approximation in this sense gives

$$c_2 = \frac{13}{8(1 + 2\log 2)} = 0{\cdot}681,$$

and $\phi_2(\tau)$ differs from the true solution $\phi(\tau)$ for τ not too large by only a few per cent. The lack of accuracy of the approximation for large τ is of no consequence for the calculation of the law of darkening (see end of § 1) owing to the decaying exponential factor under the sign of integration.

§ 3. *The Exact Value of* $\phi(0)$.

It appears to be a matter of difficulty to obtain the solution of the integral equation (11) in finite form, or at least in a series of known functions. On the other hand, it is perhaps of interest that one can obtain the exact value of the solution at $\tau = 0$, namely,

$$\phi(0) = \frac{1}{\sqrt{3}} \quad . \quad . \quad . \quad . \quad (20)$$

If we differentiate equation (11) as it stands with respect to τ, we obtain the integral equation

$$\phi'(\tau) = \Lambda(\phi') + \tfrac{1}{2}\phi(0)\mathrm{E}i(\tau) - \tfrac{1}{2}\mathrm{E}i_2(\tau) \quad . \quad . \quad (21)$$

* The physical meaning of this is that the net flux condition is preserved at the boundary $\tau = 0$. Compare with the above the numerical discussion by Milne in *M.N.R.A.S.*, **81**, 361, 1921.

for the derivative $\phi'(\tau)$. From this we deduce the logarithmic discontinuity of ϕ' at $\tau = 0$, as noted by Milne. The solution of (21) is again developable in a Neumann series,

$$\phi' = \tfrac{1}{2}\phi(0)[\mathrm{E}i + \Lambda(\mathrm{E}i) + \Lambda_2(\mathrm{E}i) + \ldots] - \tfrac{1}{2}[\mathrm{E}i_2 + \Lambda(\mathrm{E}i_2) + \Lambda_2(\mathrm{E}i_2) + \ldots],$$

which in virtue of (13*a*) gives

$$\phi'(\tau) + 1 = \tfrac{1}{2}\phi(0)[\psi_1(\tau) + \psi_2(\tau) + \psi_3(\tau) + \ldots] . \qquad (22)$$

where we have written for brevity

$$\psi_n(\tau) = \Lambda_{n-1}(\mathrm{E}i) \quad . \quad . \quad . \quad . \quad (22a)$$

To establish (20) we first multiply (22) by $\mathrm{E}i_3(\tau)$ and integrate through the range $0 < \tau < \infty$. Integrating by parts and taking account of (19) we find

$$(\phi' + 1, \mathrm{E}i_3) = (\phi', \mathrm{E}i_3) + \tfrac{1}{3} = \tfrac{1}{3} + (\phi, \mathrm{E}i_2) - \tfrac{1}{2}\phi(0) = \tfrac{2}{3} - \tfrac{1}{2}\phi(0)$$
$$= \tfrac{1}{2}\phi(0)\sum_1^\infty (\psi_\nu, \mathrm{E}i_3),^*$$

and thus

$$\tfrac{1}{2}\phi(0)\sum_1^\infty (\psi_\nu, \mathrm{E}i_3) = \tfrac{2}{3} - \tfrac{1}{2}\phi(0) \quad . \quad . \quad . \quad (23)$$

The series thus introduced may be expressed in another form. By multiplication of (14) by ψ_ν and integration through $0 < \tau < \infty$, it follows, since $\psi_{\nu+1} = \Lambda(\psi_\nu)$ and therefore $(\psi_\nu, \Lambda(\phi)) = (\psi_{\nu+1}, \phi)$, that

$$(\phi, \psi_\nu) = (\phi, \psi_{\nu+1}) + \tfrac{1}{2}(\psi_\nu, \mathrm{E}i_3),$$

and hence, summing from $\nu = 1$ to $\nu = n$,

$$(\phi, \mathrm{E}i) = (\phi, \psi_1) = (\phi, \psi_{n+1}) + \tfrac{1}{2}\sum_1^n (\psi_\nu, \mathrm{E}i_3) \quad . \quad (24)$$

We now observe that

$$(\phi, \psi_{n+1}) = (\Lambda(\phi), \psi_n) = \ldots = (\Lambda_n(\phi), \psi_1) = (\Lambda_n(\phi), \mathrm{E}i).$$

In virtue of (8*a*),

$$\Lambda_n(\phi) < \Lambda_n(1) < 1,$$

and hence

$$0 < (\phi, \psi_{n+1}) < (\mathrm{E}i, \Lambda_n(1)).$$

Since $\Lambda_n(1) \to 0$ as $n \to \infty$, as remarked earlier in § 2, we see that (ϕ, ψ_{n+1}) converges to zero. Hence from (24), on passing to the limit and taking account of (15),

$$(\phi, \mathrm{E}i) = \tfrac{1}{2}\sum_1^\infty (\psi, \mathrm{E}i_3) = 2\phi(0) - \tfrac{1}{2}.$$

If we combine this with (23) we obtain finally relation (20).

* A rigorous proof of this procedure is not difficult, but we suppress it here for the sake of brevity.

From (12) we have for the boundary temperature T_0 the exact relation

$$\frac{T_e}{T_0} = \left(\frac{16}{3}\right)^{\frac{1}{3}}.$$

For the determination of the precision of the earlier approximations we give by way of conclusion the corresponding values of T_e/T_0. The number against each author's name denotes the order of the corresponding approximation. The approximations of Jeans and Milne * are to be understood in the sense of § 2, whilst Eddington's † approximation lies in another direction.

Jeans	1	1·278.
Jeans	2	1·182.
Milne	1	1·189.
Milne	2	1·226.
Milne	3	1·232.
Eddington	2	1·230.
Exact		1·233.

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Deviations from Boyle's Law in Stellar Interiors. By H. Jones.

(*Communicated by R. H. Fowler, F.R.S.*)

Introduction.—The conditions under which an ionised gas at very high temperatures begins to show a deviation from the perfect gas laws are important in the consideration of stellar structure. It is now possible to calculate the interaction energy of two helium-like ions with fair accuracy, provided that the distance between the nuclei is not too small. When the separation is great, apart from the purely electrostatic repulsion, there will be a small attractive force between the ions. This attraction is not included in the first order perturbation calculations of the interchange theory. One expects, therefore, to obtain a fairly accurate value of the constant b in Van der Waal's equation; but no information about a. It is, however, the constant b which is primarily of interest in the stellar problem.

In § 1 the interaction energy of two helium-like ions with nuclear charge Ze is calculated, and the result checked by the determination of the diameter of the helium atom at ordinary temperatures. The screening effect of the free electrons is considered in § 2, where it is shown to be negligible until very great densities are reached, at which large deviations from the gas laws have already occurred. Finally, in § 3 an expression is obtained showing explicitly the dependence of b/V on the nuclear charge, the temperature, and the density.

* The values are taken from *M.N.R.A.S.*, **81**, 361, 1921, § 5.
† *Internal Constitution of the Stars*, § 230 (1926).

§ 1. The wave equation for two helium-like ions is

$$(1) \qquad \left[\sum_i \nabla_i^2 + 2E_0 + \epsilon + 2\left(\frac{Z}{r_{\kappa_1}} + \frac{Z}{r_{\kappa_2}} - \frac{1}{r_{12}} + \frac{Z}{r_{\lambda_3}} + \frac{Z}{r_{\lambda_4}} - \frac{1}{r_{34}}\right) + V(\overline{12}, \overline{34})\right]\psi = 0,$$

where distances are measured in units of the radius of the first Bohr orbit for hydrogen, *i.e.* a_H, and energy in units of 13·54 volts. [Hence the factor 2.] Also

$$\tfrac{1}{2}V(\overline{12}, \overline{34}) = -\frac{Z^2}{2R} - \frac{1}{r_{13}} - \frac{1}{r_{14}} - \frac{1}{r_{23}} - \frac{1}{r_{24}} + \frac{Z}{r_{\kappa_3}} + \frac{Z}{r_{\kappa_4}} + \frac{Z}{r_{\lambda_1}} + \frac{Z}{r_{\lambda_2}}.$$

$V(\overline{12}, \overline{34})$ is to be regarded as a perturbation term; it is symmetrical in the electrons 12 and 34, this being denoted by the bar above the respective numbers. R is half the distance between the two nuclei. When the two ions are widely separated $(\epsilon + V(\overline{12}, \overline{34}))$ tends to zero, and the solution of (1) is $\psi_\kappa(\overline{12})\psi_\lambda(\overline{34})$ when $\psi_\kappa(\overline{12})$ is the wave function of the helium-like ion κ in the 1^1S state.

There are six different solutions of this type corresponding to the six permutations

$$(2) \qquad I \quad (13) \quad (14) \quad (23) \quad (24) \quad (13)(24),$$

I being the identity.

If $P\psi_\kappa(\overline{12})\psi_\lambda(\overline{34})$ be one of these solutions, P being one of the six permutations (2), then the wave functions of the zero order of approximation are

$$\Psi_k = \sum_P a_k^P P\psi_\kappa(\overline{12})\psi_\lambda(\overline{34}),$$

and the coefficients a_k^P are to be determined.

In order to carry out a perturbation calculation the unperturbed wave functions must form an orthogonal set. The six functions given by (2) are not orthogonal as they stand, but by interchanging and addition such a set can be formed. It is unnecessary to do this, however, since the functions of zero order are themselves just linear combinations of the initial functions. Other elements of the determinant of the secular equation will now, however, contain the perturbation energy ϵ, besides the diagonal terms.

The remaining terms after Ψ_κ has been introduced into (1) are to be made orthogonal to each of the other solutions, thus :

$$(3) \qquad \sum_P \int \{\epsilon_k + PV(\overline{12}, \overline{34})\} a_k^P P\psi_\kappa(\overline{12})\psi_\lambda(\overline{34}) Q\psi_\kappa(\overline{12})\psi_\lambda(\overline{34}) d\tau_1 d\tau_2 d\tau_3 d\tau_4 = 0.$$

If one operates throughout by P^{-1}, and writes

$$J_{P^{-1}Q} = \int (\epsilon_k - V(\overline{12}\ \overline{34}))\psi_\kappa(\overline{12}\psi_\lambda(\overline{34})(P^{-1}Q)\psi_\kappa(\overline{12})\psi_\lambda(\overline{34}) d\tau_1 d\tau_2 d\tau_3 d\tau_4,$$

equation (3) leads in the usual way to the requirement

$$(4) \quad . \quad . \quad . \quad . \quad | J_{P^{-1}Q} | = 0,$$

when P_1 and Q_1 go separately through the six values (2).

On account of the symmetry which already exists in the simple wave functions belonging to the separated atoms, this determinant always forms linear factors.* In the case of two helium-like ions the factorisation can be carried out directly because only three of the integrals are different. Equation (4) becomes

$$(J_E - J_{(13)(24)})^3(J_E - 2J_{(23)} + J_{(13)(24)})^2(J_E + 4J_{(23)} + J_{(13)(24)}) = 0.$$

The partitions corresponding to these factors, in the order given, are

$$4 = 3 + 1 \qquad 4 = 2 + 2 \qquad 4 = 4.$$

Only the second factor therefore corresponds to a possible state of the system, the others are disallowed by the exclusion principle. The interaction energy of the two ions is therefore to the first order

$$(5) \quad . \quad . \quad . \quad \epsilon = -\frac{a - 2b + d}{\alpha - 2\beta + \delta}$$

where we have written

$$J_E = a + \epsilon\alpha$$
$$J_{(23)} = b + \epsilon\beta$$
$$J_{(13)(24)} = d + \epsilon\delta$$

The wave function of the helium-like ion in its lowest state is assumed to be

$$(6) \quad . \quad . \quad . \quad \psi_\kappa(\overline{12}) = e^{-\gamma(r_{\kappa_1} + r_{\kappa_2})}$$

apart from a normalising factor. To a first approximation $\gamma = Z$ the atomic number. The approximation will be better as Z increases. For the actual case of two helium atoms the value of γ is found which makes the assumed function agree as closely as possible with that determined by Hartree.† This method makes $\gamma = 1{\cdot}75$ for *He*. Using the functions (6) the integrals a, b, d become

$$a = \int V(\overline{12}, \overline{34}) e^{-\gamma\{2r_{\kappa_1} + 2r_{\kappa_2} + 2r_{\lambda_3} + 2r_{\lambda_4}\}} d\tau_1 d\tau_2 d\tau_3 d\tau_4$$

$$b = \int V(\overline{12}, \overline{33}) e^{-\gamma\{2r_{\kappa_1} + 2r_{\lambda_4} + (r_{\kappa_2} + r_{\lambda_2}) + (r_{\kappa_3} + r_{\lambda_3})\}} d\tau_1 d\tau_2 d\tau_3 d\tau_4$$

$$d = \int V(\overline{12}, \overline{34}) e^{-\gamma\{(r_{\kappa_1} + r_{\lambda_1}) + (r_{\kappa_2} + r_{\lambda_2}) + (r_{\kappa_3} + r_{\lambda_3}) + (\kappa r_4 + r_{\lambda_4})\}} d\tau_1 d\tau_2 d\tau_3 d\tau_4.$$

The normalising integrals α, β, δ are similar to a, b, d respectively, but with the perturbation term omitted.

From these one finds easily for (5)

$$\epsilon = \frac{Z^2}{R} - \frac{8Z(I_1I_3 - I_2I_4)}{I_1^2 - I_2^2} + \frac{4(I_5 - I_7)}{I_1^2 - I_2^2} + \frac{4(I_1^2I_5 - 2I_1I_2I_6 + I_2^2I_7)}{(I_1^2 - I_2^2)^2}$$

* London, *Zeit. für Physik.*, **50**, 24, 1928.

† Hartree, *Proc. Camb. Phil. Soc.*, 1928, **24**, 111.

The first term gives just the repulsion of the nuclei; it is to be remembered that the quantities are not measured in ordinary units.

$$I_1 = \int e^{-2\gamma r} d\tau = \frac{\pi}{\gamma^3}$$

$$I_2 = \int e^{-\gamma(r_\kappa + r_\lambda)} d\tau = \frac{\pi}{\gamma^3} e^{-2x}\left(1 + 2x + \frac{4x^2}{3}\right)$$

$$I_3 = \int \frac{e^{-2\gamma r_\lambda}}{r_\kappa} d\tau = \frac{\pi}{\gamma^2}\left\{\frac{1}{2x} - e^{-4x}\left(1 + \frac{1}{2x}\right)\right\}$$

$$I_4 = \int \frac{e^{-2\gamma(r_\kappa + r\lambda)}}{r_\kappa} d\tau = \frac{\pi}{\gamma^2} e^{-2x}(1 + 2x)$$

$$I_5 = \int \frac{e^{-2\gamma(r_{\kappa_1} + r_{\lambda_2})}}{r_{12}} d\tau_1 d\tau_2 = \frac{\pi^2}{\gamma^5}\left\{\frac{1}{2x} - e^{-4x}\left(\frac{2x^2}{3} + \frac{3x}{2} + \frac{11}{8} + \frac{1}{2x}\right)\right\}$$

$$I_6 = \int \frac{e^{-\gamma(2r_{\kappa_1} + r_{\kappa_2} + r_{\lambda_2})}}{r_{12}} d\tau_1 d\tau_2 = \frac{\pi^2}{\gamma_5}\left\{e^{-2x}\left(2x + \frac{1}{8} + \frac{5}{32x}\right) - e^{-6x}\left(\frac{1}{8} + \frac{5}{32x}\right)\right\}$$

$$I_7 = \int \frac{e^{-\gamma(r_{\kappa_1} + r_{\lambda_1} + r_{\kappa_2} + r_{\lambda_2})}}{r_{12}} d\tau_1 d\tau_2$$

$$= \frac{\pi^2}{\gamma^5}\left[\frac{3}{5x}\{S^2(\log 2x + C) + S^{*2}Ei(-8x) - 2S^*SEi(-4x)\} - e^{-4x}\left(\frac{8x^3}{15} + \frac{12x^2}{5} + \frac{23x}{10} - \frac{5}{8}\right)\right]$$

x is written shortly for γR, and $C = 0{\cdot}577216$. Euler's constant.

$$S = e^{-2x}\left(1 + 2x + \frac{4x^2}{3}\right)$$

$$S^* = e^{2x}\left(1 - 2x + \frac{4x^2}{3}\right)$$

The interaction energy can now be written

(7) . . . . . $\epsilon = \gamma . f(x, Z)$,

where

$$f(x, Z) = \frac{1}{(1 - S^2)^2}\left[\frac{(2 - Z + ZS^2)^2}{x} + e^{-4x}\left\{Z(1 - S^2)\left(\frac{64}{3}x^3 + \frac{128}{3}x^2 + 32x + 16 + \frac{4}{x}\right) - \left(\frac{1024}{45}x^3 + \frac{596}{15}x^2 + \frac{179}{5}x + 25 + \frac{29}{4x}\right)\right\} - \frac{12}{5x}(1 - 2S^2)\{S^2(\log 2x + C) + S^{*2}Ei(-8x) - 2SS^*Ei(-4x)\} - Se^{-6x}\left(\frac{256}{45}x^5 + \frac{1376}{45}x^4 + \frac{808}{15}x^3 + \frac{82}{3}x^2 - \frac{169}{15}x - \frac{31}{2} - \frac{13}{4x}\right)\right]$$

To what extent the first order energy of the perturbation calculation can be relied upon, for a system such as that considered here, can be

estimated by applying (7) to the determination of the diameter of the *He* atom at ordinary temperatures. The diameter may be defined as

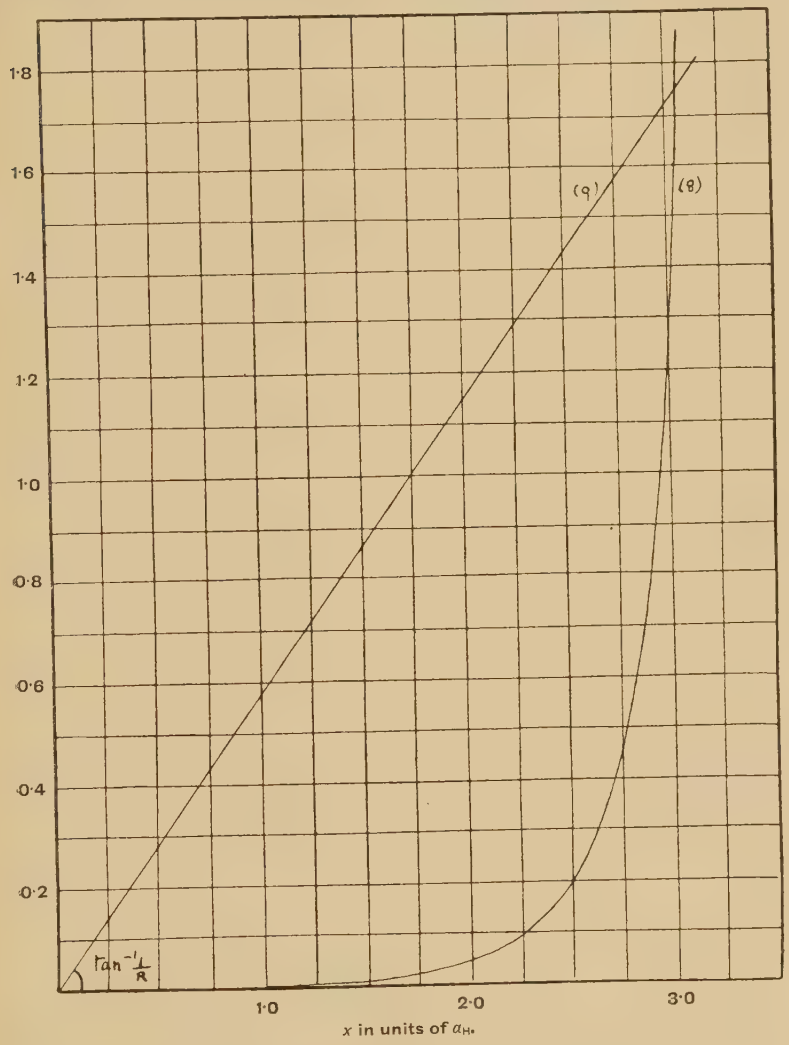

FIG. 1.—The line (9) cuts (8) at the point $\gamma = 1{\cdot}75$, the value of R given by the slope is $1{\cdot}73\, a_H$. The diameter of the atom is therefore 1·85 A.

the distance between the two nuclei when the interaction energy is equal to $\frac{3}{2}k\mathrm{T}$. It will be given by the elimination of x between

(8) . . . . $\gamma = \frac{3}{2}k\mathrm{T}/f(x, 2)$

(9) . . . . $\gamma = x/\mathrm{R}$

and determining R for the given value of γ, viz. 1·75. This is best done graphically as shown in fig. 1. The numerical values from which the graph has been drawn are given in the following table :—

x.	$f(x, 2)$.	$\{\frac{3}{2}kT/f(x, 2)\}_{T=273}$.
0·5	4·1841	..
1·0	1·0586	0·00246
1·5	0·3670	0·00711
2·0	0·0573	0·04552
3·0	0·00208	1·2539

The value of the diameter 1·85 A obtained in this way is in good agreement with the experimentally determined values at ordinary temperatures.

§ 2. In the interior of a star the positive ions are surrounded by an electron gas, which may be of great density. This may result in the screening of the nucleus, which would reduce the effective nuclear charge for the bound electrons, and therefore increase the value of γ. The primary effect of the free electrons surrounding the positive ion is to make the potential deviate from the simple inverse first power of the distance from the nucleus. The field will, however, still possess spherical symmetry. The potential function is determined by the combined use of Boltzmann's theorem and Poisson's equation, as in the Debye-Hückel theory of strong electrolytes.

The integration over the velocity co-ordinates, which is necessary to obtain the frequency ratio for the presence of an electron in a small space element, is restricted by the condition

$$\frac{m}{2}(u^2 + v^2 + w^2) > \epsilon\psi(r) ; \tag{10}$$

$\psi(r)$ is the potential at a distance r from a particular nucleus. The charge on the electron is $-\epsilon$. Equation (10) merely expresses that we are dealing with free electrons.

The average density of charge surrounding the nucleus, given as a function of the potential, which itself is an average value, is then *

$$\rho = \frac{4}{\sqrt{\pi}}\rho_0\int_{\sqrt{y}}^{\infty} e^{-t^2}t^2dt$$

$$= \frac{2}{\sqrt{\pi}}\rho_0\{\sqrt{y} + e^y\mathrm{E}rfc(\sqrt{y})\}. \tag{11}$$

In this equation $y = \dfrac{\epsilon\psi}{k\mathrm{T}}$ and ρ_0 is the density of charge at the point where $y = 0$. Poisson's equation, when there is spherical symmetry, is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dy}{dr}\right) + 4\pi a\phi(y) = 0, \tag{12}$$

where $a\phi(y)$ is written shortly for $\dfrac{\epsilon}{k\mathrm{T}}$ times the right-hand side of (11).

* Eddington, *Int. Const. of Stars*, p. 263.

The boundary conditions which y satisfies are

$$(13) \quad . \quad . \quad . \quad . \quad y \to 0 \quad \text{as} \quad r \to \infty$$
$$y \to \frac{Z\epsilon^2}{kT} \Big/ r \quad \text{as} \quad r \to 0,$$

and also one notices that as $r \to 0$, $\phi(y)$ behaves as $\sqrt{y}$. (12) may be written, using the conditions (13),

$$(14) \quad . \quad . \quad y = \int_r^\infty \frac{1}{r^2}\left[\frac{Z\epsilon^2}{kT} + 4\pi a \int_0^r \phi\{y(t)\}t^2 dt\right] dr$$

and approximate solutions can be obtained by iteration. A partial integration of (14) gives

$$y_n = \frac{Z\epsilon^2}{Tk} \Big/ r + \frac{4\pi a}{r}\int_0^r \phi\{y_{n-1}(t)\}t^2 dt + 4\pi a \int_r^\infty \phi\{y_{n-1}(t)\}t \,.\, dt.$$

y_0 given by neglecting ϕ altogether is just the potential due to the nucleus, y_1 and higher approximations take into account the free electrons surrounding the nucleus.

An upper limit can readily be calculated to the value of the average charge which is contained within a sphere of radius r_a about the nucleus. When r_a is equal to the radius of the Bohr orbit for the K electrons, this charge will give a good idea of the amount by which Z must be decreased, on account of the free electrons. This gives the effective nuclear charge, which will then be equal to γ to the approximation considered, in which the wave functions take the simple exponential form.

Each positive ion occupies on the average a volume

$$(15) \quad . \quad . \quad . \quad . \quad \frac{4}{3}\pi r_b^3 = \frac{m}{\rho_m},$$

where m is the mass of the ion and ρ_m the density of the ionised gas. Within this sphere of radius r_b there will be an average number of $(Z - 2)$ electrons; the potential and therefore y will be zero on the surface, and at every point within it will be less than that given by y_0. Therefore, since $\phi(y)$ is a steadily increasing function of y, it follows that the actual value of ϕ for given r must be less than the value given by $y_0(r)$, but greater than $\frac{\sqrt{\pi}}{2}$, which is the value of $\phi(0)$. From equations (11) and (15) it is readily seen that the amount of negative charge within a sphere of radius r_a is

$$(16) \quad . \quad . \quad \sigma(r_a) = (Z - 2)\epsilon \,.\, \int_0^{r_a} \phi(y) r^2 dr \Big/ \int_0^{r_b} \phi(y) r^2 dr.$$

The upper limit of σ is, therefore, given by taking the integral to r_a with $\phi(y)$ given by $y_0(r)$, and the integral to r_b with $\phi(y) = \frac{\sqrt{\pi}}{2}$, which is its minimum value.

The value of y_0 corresponding to r_a, which is taken to be the radius

of the K shell, is $\frac{Z\epsilon^2}{r_a}\Big/kT$. This is $\frac{3}{2}$ times the ratio of the negative energy of the bound electrons to the energy of the free electrons. If we put $\beta = \frac{Z\epsilon^2}{r_a kT}$, then clearly $\beta > \frac{3}{2}$ in order that the K electrons may remain bound. Again, if β is greater than about 6, the L electrons will not be removed. Changing the variable of integration to y, in the integral taken to r_a, (16) then gives

$$\frac{\sigma}{(Z-2)\epsilon} < 1{\cdot}309\frac{\rho_m}{w}\frac{\beta^3}{Z^3}\int_\beta^\infty \frac{\phi(y)}{y^4}dy.$$

w is the atomic weight of the ion on the oxygen scale.

For values of β which are of interest, say $\beta \geqslant 2$, the integral is easily evaluated by using the asymptotic expansion for $Erfc(x)$.

As a particular case, helium-like ions of iron may be taken. In this case $\beta = 2$ corresponds to a temperature $4{\cdot}1 \times 10^7$ degrees. Suppose the density $\rho_m = 5000$, then with $Z = 26$, $w = 56$, one finds that $\sigma < 0{\cdot}105\epsilon$. Even in a favourable case, therefore, the screening effect is negligible.

§ 3. If the constant a in Van der Waal's equation be put zero, and the equation then compared with the general equation of state,* one finds

$$(17) \quad . \quad . \quad . \quad \frac{b}{V} = -2\pi\frac{N}{V}\int_0^\infty r^2(e^{-E/kT} - 1)dr.$$

E is the potential energy of two atoms whose centres are at a distance r apart.

Equation (7) gives the interaction energy of two helium-like ions, with nuclear charge $Z\epsilon$, separated by a distance 2R in atomic units. This includes the purely electrostatic repulsion due to two positive ions of charge $(Z-2)\epsilon$. At large distances this will be much greater than the interchange energy, since it falls off inversely with the separation, whilst the latter falls off exponentially. The purely electrostatic forces have been investigated and their effects known for some time. The term E in equation (17) will be given, therefore, by subtracting the electrostatic energy $\frac{(Z-2)^2}{R}$ from (7). On rearranging the terms we have

$$(18) \quad \frac{E}{\gamma(Z-1)} = \frac{e^{-4x}\left(\frac{64}{3}x^3 + \frac{128}{3}x^2 + 32x + 16 + \frac{4}{x}\right)}{1 - e^{-4x}\left(1 + 2x + \frac{4x^2}{3}\right)^2}$$

$$+ \frac{1}{(Z-1)}\cdot\frac{1}{(1-S^2)^2}\Big[\frac{4S^2}{x} - \frac{12}{5x}(1-2S^2)\{S^2(\log 2x + C) + S^{*2}Ei(-8x) - 2SS^*Ei(-4x)\}$$

$$- e^{-4x}\left(\frac{1024}{45}x^3 + \frac{596}{15}x^2 + \frac{179}{5}x + 25 + \frac{29}{4x}\right)$$

$$- e^{-6x}S\left(\frac{256}{45}x^5 + \frac{1376}{45}x^4 + \frac{808}{15}x^3 + \frac{82}{3}x^2 - \frac{169}{15}x - \frac{31}{2} - \frac{13}{4x}\right)\Big].$$

* Fowler, *Statistical Mechanics*, p. 200.

For the important values of x which are greater than 1 the term multiplied by $\frac{1}{Z-1}$, when Z = 2, is approximately equal to the first term; when $x = 2$ it is less than half the first term. Thus, by entirely neglecting the second term the error introduced cannot be greater than $\frac{1}{Z}$; actually it will be less than this because the values of E, which mainly affect the integral (17), arise from the values of $x > 1$.

Equation (17) can now be written

$$(19) \quad \frac{b}{V} = -\frac{16\pi}{\gamma^3}\frac{a_H{}^3}{m_H}\frac{\rho_m}{w}\int_0^\infty \left(e^{-\frac{\gamma(Z-1)}{kT}g(x)} - 1\right)x^2dx,$$

where $g(x)$ is the first term of (18), a_H being the radius of the hydrogen 1 orbit, and m_H the mass of the atom of atomic weight 1. kT is measured here in units of 13·54 volts, and one notices that $\gamma(Z-1)$ is equal approximately to the energy of the K electrons in the same units. The condition that the K electrons are bound whilst the L electrons are removed is expressed approximately by

$$(20) \quad 1 < \frac{\gamma(Z-1)}{kT} < 16.$$

If T be not less than 10^7, this holds for all elements beyond iron, *i.e.* $Z > 26$. The integral in (19) has been evaluated numerically for a few values of $\frac{\gamma(Z-1)}{kT}$ lying in the range (20), and the logarithms of these values plotted against $\log\frac{\gamma(Z-1)}{kT}$. The result is almost a straight line as shown in fig. 2. In this way one finds

$$(21) \quad \int_0^\infty \left(e^{-\frac{\gamma(Z-1)}{kT}g(x)} - 1\right)x^2dx = -1{\cdot}59\left(\frac{\gamma(Z-1)}{kT}\right)^{\cdot 45}$$

within the range (20). Finally (19) becomes

$$(22) \quad \frac{b}{V} = 1{\cdot}607 \times 10^3\frac{1}{\gamma^3}\cdot\frac{\rho_m}{w}\left\{\frac{\gamma(Z-1)}{kT}\right\}^{\cdot 45}$$

As a particular example one may take helium-like ions of iron at a temperature $T = 10^7$; then the density at which a 1 per cent. deviation from the perfect gas law occurs, *i.e.* $\frac{b}{V} = \frac{1}{100}$, is $\rho_m = 469$. The upper part of the main sequence in the Russell diagram, according to Jeans's * theory of liquid centres, corresponds to a congestion of atoms ionised down to the K shell. In the empty region of the diagram below, ionisation of the K shell itself occurs, with a consequent return to the gaseous state. The ideal case in which all the atoms are ionised down to the K shell, there being no L electrons left, would correspond,

* Jeans, *Astronomy and Cosmogony*, chap. v.

therefore, to this region of the main sequence, and the theory requires that here large deviations from the perfect gas law should occur. Taking the hypothetical case which Jeans considers, viz. $\rho_m = 100$, $Z = 100$, $T = 10^8$, and one may take $w = 2{\cdot}5Z$, the value of $\frac{b}{V}$ is $1{\cdot}02 \times 10^{-5}$ according to (22), from which it would appear that the

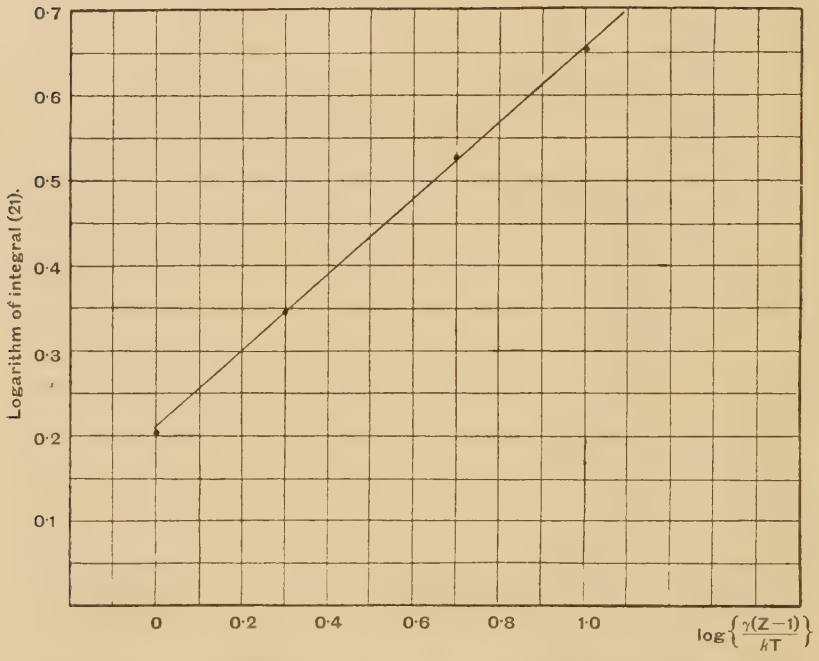

FIG. 2.—The four points marked in this diagram have been obtained by a numerical integration of (21). The graph shows that this integral can be represented approximately as the 0·45 power of the parameter $\frac{\gamma(Z-1)}{kT}$.

stellar material, as far as congestion of the ions is concerned, behaves almost exactly as a perfect gas.

Equation (18) shows that the interaction energy of two ions depends to a first approximation on the square of the effective nuclear charge, multiplied by some function of R/Z. This form for the interaction energy must be general and not merely restricted to the case of helium-like ions, as one can readily see from Mulliken's point of view, where the interaction energy in cases such as these under consideration, *i.e.* closed K and L shells, arises through the "promotion" of one or more electrons, and it is remembered, of course, that the energy of a state is proportional to the square of the effective nuclear charge. Thus, for a constant ratio of the energy of the bound electrons to the mean energy

of the free electrons, the term $\frac{b}{V}$ will vary inversely as the cube of the effective nuclear charge, as one sees from (21) and (22). This is what might be expected from a naïve consideration of atoms as pictured in the Bohr theory, and it appears, therefore, that ions with L and M shells will behave in a corresponding manner to ions containing just the K electrons, and will show no anomalously great diameters such as are required by Jeans's theory.

In conclusion I wish to thank Mr. R. H. Fowler for suggesting the work, and for his encouragement and advice.

Mean Areas and Heliographic Latitudes of Sunspots in the Year 1928.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those in the *Monthly Notices*, **89**, 264, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at the Kodaikanal Observatory. A photograph for November 23 was kindly lent for measurement by the Naval Observatory, Washington.

Table I. gives the mean daily area of umbræ, whole spots, and faculæ for each synodic rotation of the Sun included in the year 1928, whilst the mean values for each year as a whole from 1923 to 1928 are given in Table II.

Table III. gives for each rotation in the year 1928 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for both hemispheres, northern and southern. The mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots are also tabulated. The mean values for the years 1923 to 1928 are given in Table IV. Tables II. and IV. are in continuation of similar tables given in the *Monthly Notices:* for the years 1874 to 1888, **49**, 381 and 382; 1889 to 1902, **63**, 465 and 466; 1901 to 1914, **76**, 402 and 403; 1913 to 1924, **85**, 1007 and 1008.

The rotations in Tables I. and III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node of the Sun's equator on the ecliptic at mean noon on 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

The principal features of the record for the year 1928 were as follows:—

1. An increase amounting to 30 per cent. in the mean daily areas of spots and to 15 per cent. in the areas of faculæ, as compared with the corresponding measures for 1927.

2. The number of groups of spots showed a slight increase.

3. The northern and southern hemispheres were on the average about equally active.

4. The mean latitude of spots was $13\frac{1}{2}°$ as compared with 15° for the previous year; latitudes ranged from 1° to 35°. Large spots were in the mean about 4° nearer the equator in the northern hemisphere than in the southern.

5. On no days were spots and faculæ absent from the sun's disc.

6. The number of groups lasting for at least two days was 352, being equally divisible between the northern and southern hemispheres. Besides these, 126 very small groups (mostly consisting of one or two tiny spots) were measured on one day only; 56 of these were in the northern hemisphere and 70 in the southern.

7. The year 1928 was one of considerable solar activity in which the mean daily areas of spots reached a maximum for the present cycle. Table II. shows, however, that the maximum phase is represented by a double peak, the first occurring in 1926 and the second—a slightly higher one so far as the spots are concerned—in 1928. In this respect the present cycle resembles the alternate one with peaks in 1905 and 1907.

TABLE I.

No. of Rotation.	Rotation commenced G.C.T.	No. of Days on which Sun Photographed.	Mean of Daily Areas. Projected.* Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening.† Umbræ.	Whole Spots.	Faculæ.
994	Jan. 6·30	28	347	2054	1540	250	1518	1890
995	Feb. 2·64	27	271	1499	1530	208	1155	1889
996	Feb. 29·98	27	383	2308	1956	301	1820	2433
997	Mar. 28·29	28	331	1818	2155	245	1340	2548
998	Apr. 24·56	27	300	1834	2262	213	1286	2604
999	May 21·79	27	245	1246	2033	175	914	2509
1000	June 17·99	27	467	2623	2580	351	1988	3191
1001	July 15·19	28	449	2386	2474	319	1695	2992
1002	Aug. 11·41	27	286	1505	2637	222	1172	3201
1003	Sept. 7·65	27	560	3380	2489	408	2451	3071
1004	Oct. 4·93	27	263	1420	2357	184	975	2844
1005	Nov. 1·22	28	193	1125	2049	144	835	2446
1006	Nov. 28·53	27	254	1503	1788	185	1105	2180

* Expressed in millionths of the Sun's disc.
† Expressed in millionths of the Sun's hemisphere.

TABLE II.

Year.	No. of Days.		Mean Daily Areas.					
			Projected.*			Corrected for Foreshortening.†		
	Photographed.	Without Spots.	Umbræ.	Whole Spots.	Faculæ.	Umbræ.	Whole Spots.	Faculæ.
1923	365	171	15	74	189	10	55	222
1924	366	97	62	361	498	46	276	575
1925	365	19	189	1102	1508	141	830	1750
1926	365	2	284	1651	2159	217	1262	2556
1927	365	0	248	1417	1812	186	1058	2212
1928	366	0	333	1884	2134	244	1390	2589

TABLE III.

No. of Rotation.	Rotation commenced G.C.T.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean Daily Area.	Mean Heliographic Latitude.	Mean Daily Area.	Mean Heliographic Latitude.		
			°		°	°	°
994	Jan. 6·30	855	12·07	663	14·40	+ 0·51	13·09
995	Feb. 2·64	540	7·77	615	13·69	− 3·66	10·92
996	Feb. 29·98	985	11·74	835	15·60	− 8·09	13·51
997	Mar. 28·29	492	12·80	848	14·06	− 4·20	13·60
998	Apr. 24·56	233	9·83	1053	14·96	−10·47	14·03
999	May 21·79	437	10·13	477	13·70	− 2·32	11·99
1000	June 17·99	1145	12·10	844	19·76	− 1·42	15·35
1001	July 15·19	1047	11·07	649	18·16	− 0·11	13·79
1002	Aug. 11·41	613	11·93	559	13·44	− 0·16	12·65
1003	Sept. 7·65	1254	15·03	1196	15·60	+ 0·08	15·31
1004	Oct. 4·93	837	13·19	139	10·49	+ 9·82	12·80
1005	Nov. 1·22	384	13·91	451	15·21	− 1·83	14·62
1006	Nov. 28·53	760	10·07	345	12·71	+ 2·95	10·90

TABLE IV.

Year.	No. of Days on which Sun Photographed.	Spots NORTH of the Equator.		Spots SOUTH of the Equator.		Mean Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
		Mean Daily Area.	Mean Heliographic Latitude.	Mean Daily Area.	Mean Heliographic Latitude.		
			°		°	°	°
1923	365	33	13·88	22	17·32	+ 1·34	15·26
1924	366	232	22·19	45	25·54	+14·49	22·73
1925	365	517	20·70	313	19·37	+ 5·60	20·20
1926	365	663	19·27	599	17·98	+ 1·60	18·66
1927	365	379	17·75	679	13·54	− 2·33	15·05
1928	366	727	11·93	663	15·23	− 1·02	13·50

Royal Observatory, Greenwich:
1929 *December* 18.

* Expressed in millionths of the Sun's disc.
† Expressed in millionths of the Sun's hemisphere.

The Frequency of Double Stars of Different Spectral Types and Absolute Magnitudes. By Dr. E. A. Kreiken.

In a recent paper the frequency of the double stars, both the visual and spectroscopic ones, among the stars of the giant series was investigated. The material used there consisted of the stars contained in the Lick Radial Velocity Catalogue. As the limiting magnitude of that catalogue is 5·51, the number of the dwarf stars is very low. In the same article we considered the possibility of extending our results to the main sequence, by taking into account the fainter apparent magnitudes also. It appeared that with the spectroscopic doubles no satisfactory results could be expected. On the other hand an extension was possible with the visual doubles. For this we want a region of the sky which has been very thoroughly inspected for visual doubles and for which at the same time a large number of parallaxes are known. It is not necessary that all parallaxes down to a certain limit should be known, if only a selection according to proper motion is absent. The Greenwich Astrographic zone is very favourable for our present purpose. The double stars in this zone have been collected by Dr. Groot * from the Astrographic Catalogue, while a large area of this same zone is covered by the investigations of Lindblad † and Petersson,‡ who have determined the spectroscopic parallaxes of all stars measurable on their plates, without any regard to proper motion or apparent magnitude.

There can be no doubt that by collecting the double stars from an astrographic catalogue many close pairs will escape observation. If, however, we limit ourselves to stars brighter than the apparent magnitude 10·1 a large number of additional pairs may be obtained from Burnham's General Catalogue of Double Stars. It is evident that even by combining the data of both catalogues the faint double stars will statistically be less completely observed than those brighter than magnitude 5·5, as the fainter stars are less intensively observed. It is, however, possible to allow for the influence of this selection by applying certain correction factors to the observed numbers. The way in which the numerical values of these factors were derived will be explained later on.

We started by collecting all pairs from Dr. Groot's catalogue and from the β.G.C., which also occur in the catalogues of Lindblad and Petersson. Only those stars were used for which the latter authors give the absolute (photographic) magnitudes. Then the material was arranged in the following way. For each apparent magnitude the number of stars in general and the number of double stars of different

* *M.N.R.A.S.*, **88**, 51–56, 1927.

† "Spectrophotometric Determinations of Stellar Luminosities," *Nova Acta Sc. Upsala*, série iv., **6**, No. 5, 1925. "Researches Based on Determinations of Stellar Luminosities," *Ibidem*, volumen extraordinum, 1926.

‡ "The Distribution of Distances and Velocities in the Carrington Zone on the basis of Spectrophotometric Analysis," Academical Dissertation, *Meddelanden Upsala*, No. 29, 1927.

spectral types and different absolute magnitudes were counted. In order to avoid the inclusion of a large number of optical pairs a rather severe limit was adopted for the double stars: those pairs were successively considered as double stars which had (1) a magnitude difference $\Delta m < 3{\cdot}0$ and an apparent angular distance, $d < 15''$; (2) $3{\cdot}0 < \Delta m < 4{\cdot}0$ and $d < 10''$; and (3) $4{\cdot}0 < \Delta m < 5{\cdot}0$ and $d < 5''$; the pairs with $\Delta m > 5{\cdot}0$ being always excluded.

The results of these star counts appear in Table I. This table gives for the different apparent magnitudes the numbers of stars and double stars, respectively, of different spectral types and absolute magnitudes. The numbers of double stars are given in brackets.

TABLE I.

Distribution of the Total Number of Stars and of Double Stars in Absolute Magnitude and Spectral Type.

(*a*) $m = 9{\cdot}1–10{\cdot}1$.

Spectr. / $M_{abs.}$	O–B7.	B8–A5.	F.	G.	K.	M.
0·0	1 (0)	1 (0)	0	16 (0)	2 (0)	0
+1·0	1 (1)	15 (0)	2 (0)	25 (1)	4 (0)	0
+2·0	..	42 (1)	0	45 (2)	60 (0)	6 (0)
+3·0	..	79 (4)	23 (1)	27 (0)	21 (0)	0
+4·0	..	..	161 (6)	17 (0)	5 (0)	..
+5·0	..	..	58 (3)	63 (4)	1 (0)	..
+6·0	..	..	9 (1)	39 (4)	11 (0)	..
+7·0	..	..	..	16 (2)	15 (2)	..

(*b*) $m = 8{\cdot}1–9{\cdot}1$.

Spectr. / $M_{abs.}$	O–B7.	B8–A5.	F.	G.	K.	M.
0·0	1 (0)	1 (0)	0	9 (0)	22 (1)	0
+1·0	..	23 (3)	0	19 (0)	18 (2)	0
+2·0	..	65 (4)	0	18 (0)	24 (1)	6 (0)
+3·0	..	84 (3)	16 (2)	14 (0)	10 (1)	0
+4·0	..	..	79 (15)	11 (1)	1 (0)	..
+5·0	..	..	16 (1)	30 (3)	1 (0)	..
+6·0	..	..	6 (0)	12 (2)	9 (0)	..

TABLE I.—*continued.*

(*c*) $m = 7{\cdot}1-8{\cdot}1$.

Spectr. / $M_{abs.}$	O–B7.	B8–A5.	F.	G.	K.	M.
0·0	..	1 (0)	..	5 (0)	4 (0)	..
+1·0	..	22 (0)	..	6 (0)	9 (0)	..
+2·0	..	43 (5)	1 (0)	5 (1)	12 (0)	..
+3·0	..	21 (3)	6 (2)	3 (0)	7 (0)	..
+4·0	..	..	28 (5)	2 (1)	0 (0)	..
+5·0	..	..	2 (1)	5 (0)	0 (0)	..
+6·0	..	..	..	..	5 (2)	..

(*d*) $m = < 7{\cdot}1$.

Spectr. / $M_{abs.}$	O–B7.	B8–A5.	F.	G.	K.	M.
0·0	..	4 (0)	..	2 (0)	1 (0)	..
+1·0	2 (0)	33 (5)	..	5 (0)	1 (1)	1 (0)
+2·0	..	32 (2)	..	2 (0)	11 (2)	2 (0)
+3·0	..	8 (1)	2 (0)	5 (0)	4 (1)	..
+4·0	..	..	4 (0)	0 (0)	1 (0)	..
+5·0	..	..	1 (0)	3 (0)	..	..

In order to make comparable the results for the separate apparent magnitudes we have to apply correction fractions in order to allow for the relative incompleteness of our data. I think there can be no objection to assuming the frequency of the double stars among the A type stars absolutely brighter than magnitude + 3·0 to be a constant for the different apparent magnitudes. With the stars of the Lick Catalogue this frequency was found to be 15 per cent. Even if a selection according to apparent magnitude were absent we could not expect to obtain this same value from our present material, as a different criterion for a pair being a double star was used. Moreover, the absolute value of the distance between two members of a pair will vary with the mean distance of the stars and therefore also with the apparent magnitude. The result of the selection and of the variation of the true limit will be a defect of the numbers of double stars with the fainter apparent magnitudes. The amount of this defect can be found by comparing the percentages of the double stars derived from the different apparent magnitudes for the A type stars with the percentage obtained for the brighter magnitudes.

A survey of the results appears in Table II. In this table I give

the total number of A type stars up to the absolute magnitude + 3·0, the corresponding number of double stars, the percentages of the total number of stars which are double, and finally the fractions by which the observed percentages should be multiplied in order to obtain the frequency 15 per cent. For the apparent magnitudes 9·1 to 10·1 the numerical value of the correction term is based on 5 double stars only, and therefore is rather uncertain. The fraction for this range of magnitude has, however, as will appear later on, hardly any influence at all on our results.

TABLE II.

Relative Frequency and Correction Fractions of A Type Double Stars with different Apparent Magnitudes.

m Apparent.	10·1–9·1.	9·1–8·1.	8·1–7·1.	<7·1.
Total Number of Stars	137	173	87	787
Number of Double Stars	5	10	8	8
Frequency of Double Stars	3·65	5·78	9·20	10·40
Correction Fraction	4·1	2·6	1·6	1·4

TABLE III.

Correction Fractions for different Spectral Types and different Apparent Magnitudes.

Spectrum	A.	F.	G.	K.	M.
C.I	0	+0·60	+1·00	+1·60	+1·90
m.					
10·1					
	4·1	3·1	2·6	2·0	1·8
9·1					
	2·6	2·0	1·6	1·3	1·2
8·1					
	1·6	1·3	1·4	1·1	1·1
7·1					
	1·4	1·1	1·0	1·0	1·0
<7·1					

The majority of the double stars used in the present investigation were taken from the β.G.C. It will therefore be evident that the fractions in Table II. are valid for a visual grouping of the magnitudes. Lindblad and Petersson have used a photographic grouping. For the A type stars this will have no influence as their mean colour-index is almost zero. With the later types, however, the colour must have a considerable influence. If, for instance, we consider a group of stars with an apparent magnitude between the limits 9·1 and 10·1 and with a mean colour-index + 1·0, we ought to apply to the double star numbers of this group the fraction which in Table II. was derived for the interval 8·1 to 9·1. In order to obtain for the different spectral types the correct fractions, the apparent photographic magnitudes were reduced for each type separately to the visual scale by applying

a correction equal to the mean colour-index of the type in question. The fractions corresponding to these reduced magnitudes were derived in the following way. The values of Table II. were plotted against magnitude, and through the points obtained in this way a smooth curve was drawn. From this curve were read the fractions corresponding to the visual scale.

The details are given in Table III. The values of the colour-index given there were taken from the investigation by F. Seares. The magnitudes in the first column are photographic ones. By proceeding in this way the double star numbers of the earlier spectral types will be multiplied by a larger fraction than those of the later types. This, however, has no influence on the final conclusions, which will be given later on. In order to give a definite proof of this the computations were completely carried through both with the reduced values of Table III. and with the original values of Table II. This is necessary, as a number of pairs were also obtained from Dr. Groot's Photographic Catalogue. Consequently the relative incompleteness of the double star numbers with the earlier spectral types may be exaggerated by the values in Table III. Evidently the values derived from the fractions in Table II. will indicate the maximum frequency of the double stars and those from Table III. the minimum frequency.

Next the total numbers of stars of different spectral types (ordinary and double stars) between the different limits of absolute magnitude were counted. Then the correction fractions of the Tables II. and III. were applied to the observed numbers of double stars, and the total maximum and minimum numbers were determined. Evidently only a part of the main sequence is covered by our present numbers. We wish to determine the relative frequency of double stars in as many regions of the Russell-Hertzsprung diagram as possible, so it was decided to combine the results obtained here directly with those previously obtained from the data in the Lick Radial Velocity Catalogue. Therefore it is necessary to shift the curves obtained here over a certain interval, equal to the corresponding mean colour-indices given in Table III., in order to reduce them to the visual scale. The details of this reduction appear in Table IV. In this table the following data are entered for the different absolute magnitudes and the various spectral types :—

1. The total numbers of stars, including the double stars, as obtained from the Lick Catalogue ($m \leqslant 5{\cdot}5$).
2. The total numbers of stars, including the double stars, as obtained from the catalogues of Lindblad and Petersson.
3. The total numbers of double stars as obtained from the Lick Catalogue.
4. The total maximum numbers of double stars as derived from the data in the catalogues of Lindblad and Petersson.
5. The corresponding minimum numbers.
6. The total numbers of stars, all data combined.
7. The total maximum numbers of double stars.
8. The total minimum numbers of double stars.

TABLE IV.

The Distribution of Stars and Double Stars in Spectral Type and Absolute Magnitude.

(*a*) B Stars (O–B7).

Absolute Magnitude.	−4.	−3.	−2.	−1.	0.	+1.	+2.	+3.	+4.	+5.
Total numbers of stars ($m<5{\cdot}5$)			22	61	134	141	54	12	6	..	..
Total numbers of double stars ($m<5{\cdot}5$)			2	7	9	18	10	2	1	..	..

(*b*) A Stars (B8–A5).

Absolute Magnitude.	−4.	−3.	−2.	−1.	0.	+1.	+2.	+3.	+4.	+5.	+6.
Total numbers of stars ($m<5{\cdot}5$)				30	126	302	228	81	..	..	..
Total numbers of stars ($m<10{\cdot}1$)				0	7	93	182	192	..	..	..
Numbers of double stars ($m<5{\cdot}5$)				3	8	44	40	15	..	..	..
Numbers of double stars ($m<10{\cdot}1$)				0	0	14·8	25·3	30·4	..	..	..
Total numbers of stars (all data)				30	133	395	410	273	..	..	..
Total numbers of double stars (all data)				3	8	58·8	65·3	45·4	..	..	..

(*c*) F Stars.

Absolute Magnitude.	−4.	−3.	−2.	−1.	0.	+1.	+2.	+3.	+4.	+5.	+6.
Total numbers of stars ($m<5{\cdot}5$)				27	24	33	76	92	59	23	8
Total numbers of stars ($m<10{\cdot}1$)				..	1	1	23	157	172	45	21
Numbers of double stars ($m<5{\cdot}5$)				1	3	5	12	16	14	8	6
Maximum numbers of double stars ($m<10{\cdot}1$)				..	..	..	6·2	42·1	44	10·3	2·1
Minimum numbers of double stars ($m<10{\cdot}1$)				..	..	..	4·2	31·7	33·9	7·8	1·6
Total numbers of stars (all data)				27	25	34	99	249	231	68	29
Total max. numbers of double stars (all data)				1	3	5	18·2	58·1	58	18·3	8·1
Total min. numbers of double stars (all data)				1	3	5	16·2	47·7	47·9	15·8	7·7

TABLE IV.—*continued.*

(*d*) G Stars.

Absolute Magnitude.	−4.	−3.	−2.	−1.	0.	+1.	+2.	+3.	+4.	+5.	+6.
Total numbers of stars ($m<5{\cdot}5$)				27	36	62	47	24	17	27	13
Total numbers of stars ($m<10{\cdot}1$)				32	45	70	49	30	98	51	19
Numbers of double stars ($m<5{\cdot}5$)					4	6	9	6	2	8	8
Max. numbers of double stars ($m<10{\cdot}1$)				4·1	9·8	0	4·2	22·2	19·0	14·9	
Min. numbers of double stars ($m<10{\cdot}1$)				2·6	6·6	0	3·0	15·2	12·0	9·4	
Total numbers of stars (all data)				59	81	132	96	54	115	75	32
Total max. numbers of double stars (all data)			0	8·1	15·9	9	10·2	27·2	27·0	22·9	
Total min. numbers of double stars (all data)			0	6·6	12·6	9	9	17·2	20·0	17·4	

(*e*) K Stars.

Absolute Magnitude.	−4.	−3.	−2.	−1.	0.	+1.	+2.	+3.	+4.	+5.	+6.
Total numbers of stars ($m<5{\cdot}5$)				45	145	261	202	54	11	2	9
Total numbers of stars ($m<10{\cdot}1$)				44	70	182	24	5	10	20	11
Numbers of double stars ($m<5{\cdot}5$)				3	10	12	11	1	1	1	4
Max. numbers of double stars ($m<10{\cdot}1$)				5·9	5·3	4·0	2·0	0	0·8	5·7	4·9
Min. numbers of double stars ($m<10{\cdot}1$)				3·1	3·4	2·8	1·2	0	0·5	3·1	2·6
Total numbers of stars (all data)				89	215	443	226	59	21	22	20
Total max. numbers of double stars (all data)				8·9	15·3	16·0	13·0	1·0	1·8	6·7	8·9
Total min. numbers of double stars (all data)				6·1	13·4	14·8	12·2	1·0	1·5	4·2	6·6

(*f*) M Stars.

Absolute Magnitude.	−4.	−3.	−2.	−1.	0.	+1.	+2.	+3.
Total numbers of stars ($m<5{\cdot}5$)			16	66	63	2	..	
Total numbers of stars ($m<10{\cdot}1$)			1	8	..	..	..	
Total numbers of double stars ($m<5{\cdot}5$)			2	3	3	..	..	
Total numbers of stars (all data)			17	74	63	2	..	

The total maximum and minimum numbers of double stars derived in this table were expressed in percentages of the total numbers of stars. The results are given in Table V. From this table we obtain an insight into the relative frequency of the double stars in the different parts of the Russell-Hertzsprung diagram. A better survey is obtained by considering the figure 1, in which our results have been represented graphically. The black circles indicate the relative frequency of double stars for each spectral type and each absolute magnitude. The diameters of these circles have been taken proportional to the mean of the maximum and minimum percentages given in Table V. The different lines (the dotted as well as the full drawn ones) indicate lines of equal frequency. The line 5, for instance, has been drawn in such a way that all regions outside of this line have a frequency equal to or less than 5 per cent.

Table V.

Maximum and (Minimum) percentage Frequencies of Double Stars in different Regions of the Russell-Hertzsprung Diagram.

Abs. Magn. \ Spectrum.	B.	A.	F.	G.	K.	M.
−4	9·1	..	..	..	..	..
−3	11·5	..	..	..	..	..
−2	7	7	4 (4)	..	10 (7)	10?
−1	13	11	12 (12)	10 (8)	7 (7)	4
0	18	14	15 (15)	12 (10)	4 (3)	4
+1	17	16	18 (16)	9 (9)	6 (5)	..
+2	17	17	23 (19)	19 (17)	2 (2)	..
+3	..	..	25 (21)	21 (15)	9 (8)	..
+4	..	..	27 (23)	36 (28)	30 (19)	..
+5	..	..	28 (27)	69 (54)	45 (33)	..
+6						

Considering this figure we remark that the relative frequencies increase towards the left, *i.e.* with the earlier spectral types, and towards the bottom, *i.e.* with the main sequence of the dwarfs. There are several other features in the diagram which are of special interest.

Considering the equal frequency curves of our diagram, it appears that these have a pronounced maximum extension towards the right with the very bright stars. With the stars of intermediate brightness the extension is a minimum, while it increases again with the faint stars. This apparent defect of the frequencies of double stars with the intermediate magnitudes is very clearly indicated with the M, K, and G

stars. With the M stars, however, our data do not cover the dwarf sequence, only one star being observed, and we are not able to determine if here also the frequencies increase again. The intermediate magnitudes are not observed, as stars of these magnitudes do not occur at all. With the K stars, however, the observed numbers seem sufficiently large to guarantee the reality of the phenomenon. With the G stars, only the frequency for the magnitude range, + 1 to + 2, is smaller than the other ones. So we conclude that the effect is probably real. Considering the shape of the equal density curves 5 and 7, it is evident that the two large extensions of these curves towards the later types indicate the giant series and the main sequence respectively. The region of the lower frequencies apparently coincides with the gap between the giants and dwarfs. From a cosmogonical point of view this must be of importance. Let us consider, for instance, the K double stars between the limits of absolute magnitude − 1 and − 2, which have a relative frequency 10 (7) per cent. If these stars continue their development we do not expect them to pass through a region of lower frequency, unless of course the rate of development of double stars is quite different from that of ordinary stars. The different equal density curves in our diagram, therefore, probably indicate certain restrictions to which the stars are subject when passing from one state to the other. I have not tried to go into any details, as there must remain considerable uncertainties in the numerical values given in Table V. In deriving these numbers we had to adopt certain limits outside of which pairs of stars were not considered as double stars. So only a certain fraction of the double stars has been taken into account. Therefore I have not given our conclusions a more definite form.

We have to consider now the very large frequencies of the double stars with the fainter magnitudes of the main series. There is a possibility that these frequencies are exaggerated. Even with the small limits of Δm adopted here, a relatively large number of double stars with a small angular distance, d, will escape observation. Let us suppose, for instance, that the pairs with $d = d_1$ are not observed. Then with the fainter absolute magnitudes the absolute distance corresponding to this value of d_1 will decrease. In the *B.A.N.*, 178,* we have considered the distribution curve $\psi(L)$ of the angular distances $L = \log d$. From a preliminary solution we obtained

$$\psi(L) = \text{const. } 10^{-h^2(L-L_0)^2},$$
$$h = 0{\cdot}473,$$
$$L_0 = 0{\cdot}64.$$

Apparently the numerical value of $\psi(L)$ increases with the smaller distances, while a maximum is found for

$$L = 0{\cdot}5 \quad \text{or} \quad d = 3{\cdot}16 \qquad d = 1'' = 66 \text{ astr. units.}$$

Owing to the scantiness of our material only a crude estimation can be made as to the effect of the variation of the true value of L with

* *B.A.N.*, **5**, 1929, p. 13, table 7.

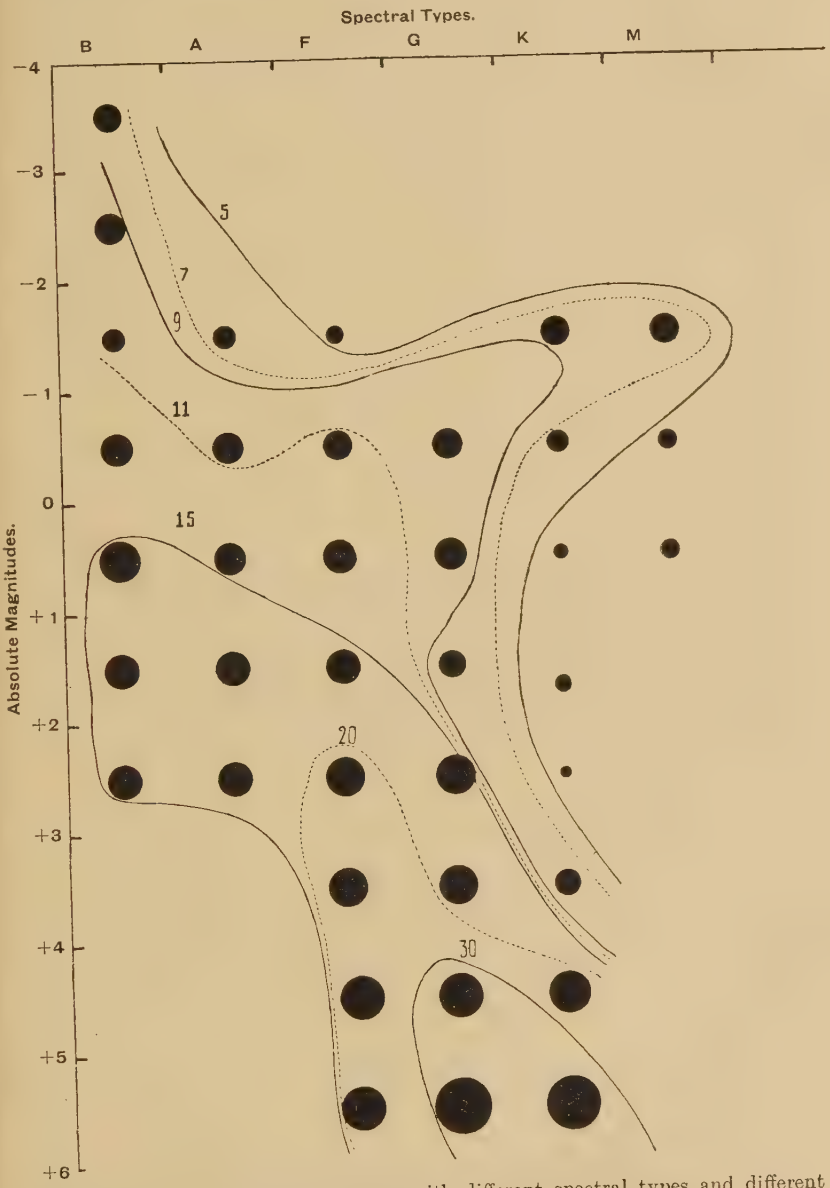

FIG. 1.—Frequencies of double stars with different spectral types and different absolute magnitudes. The diameters of the black dots are proportional to the relative frequencies. The curves are lines of equal frequency.

the fainter absolute magnitudes on our double star numbers. The large majority of our stars consist of pairs with apparent angular distance $d < 15''$ and a magnitude difference $\Delta m < 3{\cdot}0$, so I tried to determine the fractions of the total numbers of double stars within the same magnitude interval and with an apparent angular separation between the limits d_1 and d_2, where d_2 was always taken as $15''$, while for d_1 the values $0''{\cdot}25$, $0''{\cdot}50$, and $1''{\cdot}00$ respectively were adopted. Evidently the numerical values of the fractions will be equal to

$$\frac{1}{\sqrt{\pi}}\int_{\hbar L_1}^{\hbar L_2} e^{-\frac{0{\cdot}473^2}{\text{mod.}}(L-L_0)^2} dL \quad . \quad . \quad . \quad (2)$$

As stated before, L_0 will be $0{\cdot}64$ for a distance 66 parsecs. For any other distance we have

$$d_0 \times \frac{r}{66} = \text{const.},$$

$$\log d_0 + \log \frac{r}{66} = 0{\cdot}64,$$

$$\bar{L}_0 = 0{\cdot}64 - \log r + \log 66.$$

In order to obtain the values of r we determined the mean apparent magnitude $\bar{m}$ for the different intervals of absolute magnitude, and from the differences $M - \bar{m}$ we obtained the mean distances $\bar{r}$ given in Table VI. The corresponding values of L_0 are contained in the next column. The numerical values of the formula (2) for the different limits of d appear in the following three columns. Finally, we computed the fractions by which the observed numbers of double stars ought to be multiplied in order to reduce them to the interval, 0 to + 1.

TABLE VI.

Influence of the Mean Distance on the Observed Fractions.

$M_{abs.}$	$\bar{r}$.	L_0.	Observed Fraction.			Reduction to 0—+1.		
			$d_1=0''{\cdot}25$.	$d_1=0''{\cdot}50$.	$d_1=1''{\cdot}00$.	$d_1=0''{\cdot}25$.	$d_1=0''{\cdot}50$.	$d_1=1''{\cdot}00$.
−2								
	270	+·03	·622	·513	·391	1·02	1·07	1·13
−1								
	160	+·26	·639	·544	·431	1·00	1·01	1·02
0								
	125	+·36	·637	·549	·441	1·00	1·00	1·00
+1								
	96	+·48	·628	·550	·440	1·01	1·00	1·00
+2								
	80	+·56	·623	·552	·452	1·02	1·00	0·98
+3								
	84	+·54	·622	·548	·451	1·02	1·00	0·98
+4								
	51	+·75	·645	·547	·505	0·99	1·00	0·88
+5								
	22	+1·12	·484	·444	·497	1·32	1·24	0·89
+6								

From Table VI. it appears that the variation of the absolute values of the apparent limits of L with absolute magnitude does not materially influence our results, so apparently the large frequency of the double stars among the faint stars is real, though owing to the rather small numbers of stars there may remain some uncertainties as to the exact numerical values. Concerning the physical interpretation of this phenomenon I will only remark that it does not prove either that new double stars are formed along the main series also or that the rate of the development of a double star is different from that of an ordinary star. Previously it was shown that the frequency of the spectroscopic doubles is very large among the early types of the main series. When progressing along the main series these spectroscopic doubles might be converted into visual doubles by a widening of their orbits. Anyhow the present material does not allow a definite conclusion.

In the present paper also the results derived by us are quite different from those obtained by Öpik. Öpik has designed a very ingenious method for determining the coefficient of perception for the different groups of double stars. His values of the relative frequencies are, however, not directly based on parallax measures, and this probably is the chief cause of the discrepancies.

The main results of the present paper are summarised in the following points :—

1. The relative frequencies of the double stars of different spectral types and different absolute magnitudes have been determined.

2. From the diagram in which these relative frequencies are graphically represented it appears that they increase from the right to the left and towards the bottom of the figure. Along the main sequence of dwarfs the frequencies are all larger than outside of it.

3. If in the same diagram the lines of equal frequency are drawn, it appears that the shapes of these lines of equal frequency indicate the giant series as well as the main series. In the regions where the gap between the two groups of stars is to be expected the frequencies are very low.

4. The lines of equal frequency probably indicate certain restrictions to which the stars are subject when developing.

5. The influence of selection, owing to the change of the absolute value of the separation of the components corresponding to the apparent limit d, was found to be negligible.

Micrometrical Measures of Double Stars (22*nd Series*).

By Rev. T. E. Espin and W. Milburn.

The micrometrical measures are given as in previous lists, those marked E being made with the 24 in. and the ones marked M with the $17\frac{1}{4}$ in.

Micrometrical Measures of Double Stars (22nd Series).

Name.	1900. R.A.	Decl.	P.	D.	Mags.	Nts.	Date.	
	h m	° ′	°	″				
A.C. 1	0 15·6	+32 26	283·5	1·22	6·0 6·5	2	1927·991.	E.
Es. 319*	1 22·9	33 1	284·6	2·16	9·5 9·7	1	1927·903.	E.
Barton 7	2 32·3	28 41	186·4	4·52	10·0 10·4	3	1928·047 BC.	M.
Es. 324	33·2	28 27	16·3	2·25	C=12·0	3	1928·071 BC.	M.
			183·2	..	9·3 9·4	2	·062 AB.	M.
Σ 427	3 34·5	28 27	206·3	6·86	6·5 7·0	2	1927·079.	M.
Es. 424	8 0·6	35 15	72·9	5·74	9·0 11·0	2	1928·198.	E.
Barton 23*	10 5·2	29 4	188·9	4·29	9·5 10·5	1	1927·241.	M.
Σ 1458	33·8	32 13	215·9	17·82	8·2 8·4	1	1927·323.	E.
h 2543*	38·5	33 1	34·9	2·64	9·3 9·6	3	1927·286 AB.	E.
			263·2	19·05	C=13·2	3	·286 AC.	E.
h 2577	11 29·2	28 19	184·0	4·40	9·2 10·5	3	1927·292.	M.
Σ 1651	12 26·7	27 34	212·3	6·74	9·0 9·5	1	1928·309.	M.
Σ 1700	53·8	27 40	85·6	6·97	8·5 9·0	1	1928·309.	M.
Σ 2381	18 41·6	28 9	123·8	8·65	8·4 9·6	2	1927·652.	M.
Hu. 935	42·9	32 4	155·2	3·01	9·3 9·4	2	1929·708.	E.
Ho. 447*	19 12·2	27 45	174·3	2·76	9·5 9·8	2	1929·643.	M.
OΣ 376	31·4	33 59	228·2	3·00	8·0 10·5	1	1927·690.	E.
Barton 42	43·4	28 23	311·3	4·48	9·3 9·4	2	1927·713.	M.
J. 1559	45·8	33 10	85·2	3·98	9·5 13·5	3	1928·847 AB.	E.
			258·0	7·68	C=13·0	2	·836 AC.	E.
Es. 495	54·3	28 43	243·6	..	8·5 9·8	2	1927·699 AB.	M.
			313·5	4·36	9·9 10·6	3	·705 BC.	M.
Es. 2350	20 2·9	33 11	240·7	4·41	12·0 13·0	3	1929·862 BC.	E.
Barton 47	10·6	27 49	232·4	3·66	9·3 9·4	3	1927·813 AB.	M.
			211·6	30±	C=11·0	1	·791 AC.	M.
Barton 49	30·0	27 53	113·4	3·64	10·0 10·2	4	1929·713 AB.	M.
			197·5	9·19	C=10·0	1	·682 AC.	M.
Es. 510	43·9	27 31	230·8	6·36	10·5 10·8	3	·820 BC.	M.
			343·6	..	A= 8·0	2	·814 AB.	M.
h 1609*	59·8	28 17	33·7	8·04	9·3 9·4	2	1927·758 AB.	M.
			194·8	4·88	13·5 13·8	4	·768 CD.	M.
Sch. 1446	21 8·9	32 24	159·1	7·87	9·4 9·5	1	1928·906.	E.
h 1632	13·7	28 0	42·6	8·19	9·4 9·4	2	1928·784.	M.
Es. 377	18·1	31 13	210·7	2·66	9·7 10·7	1	1929·862 BC.	E.
Barton 57	35·4	29 2	138·9	4·56	9·3 9·4	2	1927·699.	M.

Name.	1900. R.A. h m	Decl. ° ′	P. °	D. ″	Mags.		Nts.	Date.	
Es. 382*	21 46·6	+32 11	314·0	10·68	7·5	14·0	3	1927·784	AB. E.
Es. 527*	56·8	27 22	212·5	3·33	..		2	1929·782.	M.
h 1760	22 19·6	26 41	338·5	6·24	9·3	9·8	3	1929·882.	M.
Es. 2269	27·0	32 4	67·6	4·04	10·0	13·5	1	1929·950.	E.
Es. 541*	35·9	26 13	222·0	7·58	8·7	11·0	2	1929·752.	M.
h 969*	41·3	33 27	29·2	6·40	8·5	9·0	3	1929·928.	E.
h 1817*	46·3	33 56	236·8	10·68	9·0	9·5	3	1929·923.	E.
Σ 2967	54·2	27 12	4·6	7·05	8·6	9·3	2	1927·884.	M.
Es. 398	23 19·8	31 45	261·5	5·87	9·6	11·0	2	1930·001.	E.
Es. 2207	30·5	34 8	241·7	7·22	8·5	10·0	2	1929·964.	E.
Es. 403	35·7	30 34	296·1	2·87	9·5	9·8	1	1929·997.	E.
h 989*	38·8	32 53	279·9	11·18	10·1	12·0	4	1929·954.	E.
h 995*	53·8	28 6	41·4	4·06	9·0	10·0	2	1929·887.	M.

Notes.

Es. 319.	$1^h\ 22^m{\cdot}9$.	Observed under poor conditions.
Barton 23.	$10^h\ 5^m{\cdot}2$.	Previously noted as double at Oxford, see vol. viii, No. 1, p. 105, under No. 5238.
h 2543.	$10^h\ 38^m{\cdot}5$.	No other measures of C.
Ho. 447.	$19^h\ 12^m{\cdot}2$.	Large increase in distance and small decrease in angle, see Greenwich Double Stars 1893 to 1919.
h 1609.	$20^h\ 59^m{\cdot}8$.	CD noted by β in 1902, but not measured.
Es. 382.	$21^h\ 46^m{\cdot}6$.	AC. P. 321°·2, 1 nt., 1927·796.
Es. 527.	$21^h\ 56^m{\cdot}8$.	Increase in angle and distance.
Es. 541.	$22^h\ 35^m{\cdot}9$.	Increase of distance ?
h 969.	$22^h\ 41^m{\cdot}3$.	Motion.
h 1817.	$22^h\ 46^m{\cdot}3$.	Angle decreasing.
h 989.	$23^h\ 38^m{\cdot}8$.	No other measures.
h 995.	$23^h\ 53^m{\cdot}8$.	Motion.

New Double Stars.

Espin No.	B.D. °	1900. R.A. h m	Decl. ° ′	P. °	D. ″	Mags.		Nts.	Date 1929.	
2363	+30·115	0 41·1	+30 48	30·3	2·93	9·5	9·6	4	·976	
2364	+30·126	48·3	30 27	133·4	7·61	9·0	12·5	3	·992	
2365	+31·189	1 4·8	32 12	65·8	5·95	9·0	11·0	2	·995	
2366	..	58·0	33 20	85·4	7·43	10·2	10·5	2	·901	(1930)
2367	+31·3462	19 2·5	31 49	319·8	6·55	9·2	9·4	2	·719	
2368	+31·3496	7·9	32 2	298·7	4·08	9·0	12·5	2	·679	
2369	+31·3628	26·6	31 47	98·6	7·82	9·2	13·0	2	·742	
2370	+32·3555	42·3	32 25	97·7	3·44	9·5	9·6	2	·679.	
2371	..	19 46·1	32 28	256·4	7·93	10·0	11·5	3	·779	AB
				168·2	2·65	12·0	13·0	4	·788	CD
				127·8	16·82	..		4	·800	AC

Espin No.	B.D.	1900. R.A.	Decl.	P.	D.	Mags.	Nts.	Date 1929.	
	°	h m	° ′	°	″				
2372	..	19 47·1	+32 21	127·8	2·49	11·0 12·0	3	·741	
2373	..	20 2·3	33 11	107·1	4·64	10·0 13·5	3	·830	AB
				60·2	7·57	C=13·0	3	·830	AC
2374	+31·4038	16·0	31 4	101·2	9·80	9·5 11·0	2	·872	
2375	+31·4095	24·0	32 0	14·0	6·89	8·5 12·0	2	·812	
2376	+32·3912	39·7	32 21	124·0	6·74	9·3 11·0	2	·821	
2377	..	44·1	33 4	207·4	3·45	10·8 11·0	2	·872	
	+33·4033	44·7	33 11	83·8	24·86	8·5 9·5	1	·871	
2378	..	46·3	33 15	18·2	5·91	10·5 10·8	3	·866	
2379	+31·4251	49·0	31 58	123·8	6·58	9·0 12·0	2	·857	
2380	..	49·3	31 57	321·9	5·33	10·7 10·9	2	·857	
2381	+31·4283	53·6	31 48	102·3	3·88	9·5 13·0	3	·858	
2382	..	21 22·6	33 11	235·9	5·65	..	2	·934	
2383	..	28·0	33 14	35·2	5·90	9·8 12·0	2	·921	
2384	..	28·7	33 14	40·7	6·37	10·5 11·0	2	·884	
2385	+32·4245	40·6	32 54	265·4	4·74	9·3 14·0	3	·893	
2386	..	44·4	32 23	230·2	5·66	9·8 11·0	3	·901	
2387	..	55·9	31 59	312·6	6·60	10·0 10·0	3	·950	
2388	+32·4356	22 6·5	33 3	121·2	6·08	9·4 13·3	3	·815	
2389	+35·4772	16·7	35 14	26·8	6·86	9·2 11·5	3	·987	
2390	..	19·4	31 56	321·2	7·01	10·5 10·7	2	·953	
2391	..	21·7	33 13	55·5	4·58	12·0 12·0	3	·944	
2392	+34·4709	27·9	34 44	46·7	6·91	9·5 12·0	3	·987	
2393	..	33·3	34 35	209·3	5·28	11·0 11·0	3	·959	
2394	..	35·1	34 43	315·9	5·10	10·2 11·5	2	·953	
2395	..	42·8	32 54	333·7	7·01	11·0 11·0	3	·980	
2396	..	47·5	32 17	153·9	5·08	9·6 10·2	1	·996	
	+34·4814	55·3	34 35	264·3	11·67	9·4 10·5	3	·939	
	+32·4611	23 11·1	32 39	105·7	13·87	9·3 11·0	1	·846	
2397	..	31·3	34 6	317·2	5·14	9·7 12·2	4	·973	
2398	..	37·4	32 35	272·8	8·72	9·7 9·9	2	·898	
2399	+34·4991	37·7	34 31	72·4	6·29	9·6 10·7	2	·990	
2400	..	58·1	32 42	104·6	6·10	10·5 10·5	3	·961	

Notes.

2366. 10 sec. P., $4\frac{1}{2}'$ N. of Ho. 3.
2385. S. star of pair.
2391. 20 sec. P., B.D. +32°·4429.
2392. Two *comites* N.
2396. Some doubt as to place.
2398. S.P. star of a triangle.

Milburn No.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1929.	
	°	h m	° ′	°	″				
592	+25·21	0 9·6	+26 6	237·9	2·57	9·5 9·6	2	·957	
593	..	10·4	27 14	167·4	6·47	9·6 12·7	3	·964	
594	..	39·0	27 1	52·4	3·78	10·0 12·0	3	·983	
595	+27·138	47·3	27 24	302·9	6·17	9·5 10·7	3	·955	
596	..	48·2	27 24	353·1	3·48	10·0 10·5	3	·859	
597	..	58·3	30 10	89·3	3·75±	11·0 11·1	2	·991	
598	..	1 16·4	27 14	86·9	5·48	10·0 12·5	3	·983	
599	..	59·5	28 38	175·2	2·37	10·0 12·0	2	·960	
600	..	2 1·7	27 42	243·1	5·13	9·5 10·0	1	·005 (1930)	
601	+27·3309	19 11·7	27 43	265·6	4·71	9·5 10·0	2	·643	
602	..	37·1	27 58	149·6	6·03	10·5 11·5	3	·741	
603	+27·3526	45·2	27 11	30·4	4·94	9·5 9·9	2	·643	
604	..	55·8	27 18	258·4	4·29	10·5 13·0	2	·679	
605	+27·3579	57·0	27 42	176·9	5·82	9·0 10·5	2	·727	
606	..	20 1·2	27 39	39·7	6·74	10·6 10·8	2	·814	
607	..	4·8	27 0	33·2	4·48	9·6 10·5	2	·784	
608	+27·3669	12·5	27 47	226·4	8·92	9·5 9·8	2	·643	
609	..	15·3	27 21	278·7	4·52	10·0 10·0	2	·742	AB
				286·5	13±	C=11·0	2	·742	BC
610	..	15·4	27 8	307·7	6·01	10·0 10·5	2	·719	AB
				206·4	65±	C= 7·5	1	·717	AC
611	+27·3708	18·0	27 17	170·4	4·93	9·0 11·5	3	·786	
612	..	34·0	27 45	169·6	6·28	9·4 9·5	2	·727	
613	..	35·7	27 32	147·3	7·46	9·6 11·7	3	·774	
614	+27·3923	51·6	27 41	93·9	10·69	9·5 10·2	2	·679	
615	..	51·9	27 53	193·1	4·02	10·0 11·5	2	·742	
616	+27·3937	55·3	27 17	230·5	3·73	9·4 11·5	3	·820	
617	..	55·8	27 38	335·7	5·26	10·1 13·7	3	·846	
618	..	55·8	27 34	340·9	4·93	10·1 13·2	3	·847	
619	..	57·1	27 44	48·2	4·40	10·2 10·2	2	·868	
620	+26·4210	21 31·6	26 49	238·5	5·93	9·0 10·0	2	·864	
621	..	32·3	26 34	46·8	3·13	9·9 10·0	3	·870	
622	..	22 0·0	27 29	274·5	3·91	11·5 13·0	2	·864	
623	+26·4432	21·4	27 7	302·7	9·57	9·4 9·7	2	·906	
624	..	31·7	29 36	110·6	2·22	10·2 12·2	2	·953	
625	..	52·1	28 38	133·3	6·93	10·0 10·0	2	·752	
626	..	54·2	26 24	138·9	2·98	10·0 10·2	3	·846	
627	..	23 3·3	26 38	247·3	5·55	10·2 10·4	2	·814	

Milburn No.	B.D.	1900. R.A.	Decl.	P.	D.	Mags.	Nts.	Date 1929.
	°	h m	° ′	°	″			
628	+29·4947	22 27·6	+29 38	327·8	2·10	9·5 9·7	3	·964
629	..	28·6	27 7	38·7	2·96	11·0 12·5	5	·945
630	+25·5024	46·8	26 15	295·1	2·37	9·5 12·5	5	·959

B.D. Stars Missing or Faint.

		h m s	° ′			
1	+30°5	0 2 24·9	+30 56·9	Mag. 9·5 (1855)	Dec. 17, 1929.	Not seen in moonlight. E.
2	+26·4435	22 20 39·9	26 1·7	Mag. 9·5 (1855)	Oct. 26, 1929.	Missing. M.
3	+25·4829	22 44 42·3	25 47·1	Mag. 9·5 (1855)	Oct. 25, 1929.	Missing. M.
4	+29·4955	23 26 24·5	29 45·4	Mag. 9·5 (1855)	Dec. 27, 31, 1929.	Faint. M.

Notes.

2. A star, 11·0 mag. N. and another NF. the place. M.
3. No star above 12 mag. near place, same Oct. 26. M.
4. 11 mag. NP. place. M.

Revision of the Elements of Comet Pons-Coggia-Winnecke-Forbes at the Apparitions of 1873 *and* 1818. By A. C. D. Crommelin.

The improvement of the elements of this comet at the apparition of 1928, which was effected with the aid afforded by M. Quénisset's early photograph, suggested to me to take similar action with regard to the two other observed apparitions. M. Schulhof's elements with period 27·91 years were not definitive, but were deduced differentially from those obtained using the period 55·82 years, which he regarded as more probable. I have taken the period as 28·0664 years; this is based on my figures in *M.N.*, **89**, 560, 1929, giving double weight to the second pair of revolutions, and is unlikely to be in error by more than a few days. It is obviously futile to attempt to improve the period from the 1873 observations, so I assumed the above value, and obtained a solution for the remaining elements from the observations, which are given in full by M. Schulhof in *B.A.*, **3**, 126, 1886. The resulting elements are given below, also the revised elements for the apparition of 1818; the change in these arises partly from the change in the 1873 orbit, partly from a slight revision of the perturbations in the interim :—

T in G.M.T.	1818 Feb. 6·05	1873 Dec. 1·95949.
	° ′ ″	° ′ ″
ω	195 43 58	196 7 45·6
Ω	249 29 29	249 49 13·2
i	29 8 7	28 46 31·2
ϕ	66 43 21	66 47 54·2
log q	9·872593	9·8732764
Period (years)	27·731	28·0664
Equinox	1818·0	1873·0

The residuals for the separate days in 1873 (Observed—Computed) are as follows :—

	R.A.	Decl.	No. of Obsns.
	″	″	
Nov. 11	−9·16	−5·13	2
12	+2·30	+2·51	11
13	+2·27	+0·06	6
14	−2·04	+0·61	3
15	−3·03	+0·38	1
16	+7·12	−1·67	1

I have followed M. Schulhof in applying a correction of $-0^s\cdot70$ to the Strassburg Right Ascensions, derived from comparison between them and those of all the other observatories. The Strassburg observations are of special importance, since one of the two observations on Nov. 11, and the only one on Nov. 16, were made there. The process used may leave a systematic error of R.A. in the observations, but its effect on the elements would not be serious. The equations of Nov. 12, 13, 14 were given double weight, owing to the larger number of observations on them. It should be noted that the mean distance of the comet from the sun during the observations was 0·822, and from the earth 0·224, so that the heliocentric errors are much smaller than the geocentric ones.

In revising the 1818 observations I formed new values of the sun's rectangular co-ordinates from the data given in *Viertelsjahrsschrift der Astr. Gesells.*, **5**, 167, 1870 ; and I included the reduction from the earth's centre to Marseilles ; the mean corrections to my previous values of log X, log Y, log Z (*M.N.*, **89**, 561, 1929) are + 3, − 10, − 12, in units of the fifth decimal. The values of T now become Feb. 5·783 from the declination on Feb. 23 ; Feb. 6·357 from the R.A. on Feb. 26 ; Feb. 5·987 from the distance on Feb. 27, assuming that Pons used Lalande 4976 as his comparison star on that day ; I adopted it instead of τ^1 Eridani, on the grounds : (1) that Pons stated that the comet was *east* of the star ; (2) that it seems unlikely that he would describe the comet simply as 50′ east of the star, if it were at the same time more than 100′ north of it, as was the case with τ^1 Eridani ; (3) Pons definitely noted the magnitude of the star used on Feb. 26 as 4, but he made no note about the magnitude of that used on Feb. 27, so that it was probably fainter. Moreover, I have now assumed that the 50′ was the whole distance between the comet and the star, not merely its component in an E.W. direction ; it would be much simpler and more natural to estimate the whole distance rather than its resolved part in a certain direction.

The simple mean of the three values for T is Feb. 6·05 ; it does not seem probable that its uncertainty can ever be reduced below 0·25 day, in view of the roughness of Pons's records. Adopting this value of T, the calculated declination on Feb. 23 is 14° 52′ S., which is 23′ north of the recorded value ; the calculated R.A. on Feb. 26 is 36° 55′·7, which is $18\frac{3}{4}'$ greater than the recorded value ; the calculated distance of the

comet from Lalande 4976 on Feb. 27 is 45′ (5′ less than the recorded value), the comet being 40′ (of great circle) east of the star, and 21′ south of it. The uncertainty of the elements, other than T, in 1818 could be considerably reduced by using shorter time-intervals in the computation of the perturbations between 1818 and 1873.

Addendum.—I have made a recomputation of the perturbations of the comet from 1928 to 1956, and obtain

$$\delta i - 244''{\cdot}5, \quad \delta\Omega + 157''{\cdot}5, \quad \delta\phi + 30''{\cdot}8, \quad \delta\omega + 362''{\cdot}4,$$
$$\delta n + 0''{\cdot}2931, \quad \text{T 1956 October } 3{\cdot}2. \text{ U.T.}$$

Determination of the Parameter of the Orbit of a Heavenly Body from Three Geocentric Distances. By G. M. Bazhenov.

(*Communicated by the Secretaries.*)

If the geocentric distances ρ_i for three observations at the times t_i ($i = 1, 2, 3$) are known, one may find from the formulæ

$$x_i = \lambda_i\rho_i - \mathrm{X}_i; \quad y_i = \mu_i\rho_i - \mathrm{Y}_i; \quad z_i = \nu_i\rho_i - \mathrm{Z}_i; \quad r_i^2 = x_i^2 + y_i^2 + z_i^2,$$

the heliocentric rectangular co-ordinates x_i, y_i, z_i and the radii-vectores r_i of the heavenly body; where

$$\lambda_i = \cos\delta_i \,.\, \cos\alpha_i; \qquad \mu_i = \cos\delta_i \,.\, \sin\alpha_i; \qquad \nu_i = \sin\delta_i,$$

and α_i and δ_i are its geocentric right ascension and declination.

Knowing the values x_i, y_i, z_i, r_i, one may compute the values of

$$s_i = r_kr_j - x_kx_j - y_ky_j - z_kz_j = 2r_kr_j \,.\, \sin^2 f_i;$$
$$u_i = r_kr_j + x_kx_j + y_ky_j + z_kz_j = 2r_kr_j \,.\, \cos^2 f_i;$$

$$n_i = \sqrt{s_i \,.\, u_i} = r_kr_j \,.\, \sin 2f_i; \quad \mathrm{S}_i = s_k + s_j - s_i; \quad i, j, k = 1, 2, 3;$$
$$\mathrm{T} = s_1\mathrm{S}_1 + s_2\mathrm{S}_2 + s_3\mathrm{S}_3 = \mathrm{S}_1\mathrm{S}_2 + \mathrm{S}_2\mathrm{S}_3 + \mathrm{S}_3\mathrm{S}_1; \qquad i \neq j \neq k;$$
$$\xi_i = \tan g_i = (-1)^{i-1}\sqrt{\mathrm{T}}/\mathrm{S}_i \quad . \quad . \quad . \quad (1)$$

where $2f_i$ and $2g_i$ are the differences of the true and eccentric anomalies of the heavenly body at the moments t_k and t_j.

These formulæ (1) for ξ_i are obtained from the known relations $\sqrt{s_i}/\sin g_i = \sqrt{2ap}$, where a and p are the semi-transverse axis and the parameter of the orbit of the heavenly body. These formulæ are valid for any conic section; but for the ellipse $\mathrm{T} > 0$ and ξ_i are real values; for the parabola $\mathrm{T} = \xi_i = 0$; for the hyperbola $\mathrm{T} < 0$ and ξ_i are imaginary.

The value $\eta_i = \dfrac{\text{sector}}{\text{triangle}} = \dfrac{k\tau_i\sqrt{p}}{n_i}$, where $k = 0{\cdot}01720210$, is the constant of Gauss, and $\tau_i = (t_k - t_j) - 0{\cdot}005770(\rho_k - \rho_j)$ may be determined from the known equation

$$\eta_i^3 - \eta_i^2 = m_i \,.\, \mathrm{G}(\xi_i) \quad . \quad . \quad . \quad (2)$$

where
$$m_i = \tau_i^2/u_i^{\frac{3}{2}},$$
and
$$G(\xi) = \frac{k^2}{2\sqrt{2}} \cdot \frac{2g - \sin 2g}{\sin^3 g} = \frac{k^2}{\sqrt{2}} \cdot \frac{(1+\xi^2)^{\frac{3}{2}}}{\xi^3} \cdot \left(\arctan \xi - \frac{\xi}{1+\xi^2}\right)$$
$$= \frac{k^2\sqrt{2}}{3} \sum_{\nu=0}^{\infty} a_\nu \xi^{2\nu} = \frac{k^2}{\sqrt{2}} \cdot \frac{(1-\xi'^2)^{\frac{3}{2}}}{\xi'^3} \cdot \left(\frac{\xi'}{1-\xi'^2} - \frac{1}{2} ln \frac{1+\xi'}{1-\xi'}\right),$$
where
$$\xi' = \xi/\sqrt{-1} \quad \text{and} \quad a_\nu = \sum_{\mu=0}^{\mu=\nu} \frac{3\mu+3}{2\mu+3} \binom{\nu}{\mu}.$$

Table I.

	$G(\xi)\cdot 10^9$					
ξ.	0.	2.	4.	6.	8.	
0·00	139494	494	495	496	497	1
1	498	500	503	505	508	3
2	511	515	518	523	527	5
3	532	537	543	549	555	6
4	561	568	575	583	591	8
0·05	139599	607	616	625	635	10
6	645	655	665	676	687	12
7	699	711	723	735	748	13
8	761	775	789	803	817	15
9	832	847	863	878	894	17
0·10	139911	928	945	962	980	18
11	998*	016*	035*	054*	073	20
12	140093	113	133	154	175	21
13	196	218	239	262	284	23
14	307	330	354	378	402	24
0·15	140426	451	476	501	527	26
16	553	579	606	633	660	28
17	688	716	744	772	801	29
18	830	860	889	919	950	30
19	980*	011*	042*	074*	106	32

Table II.

	$G(\xi')\cdot 10^9$					
	0.	2.	4.	6.	8.	
	139494	494	494	493	492	− 2
	490	488	486	484	481	− 3
	478	474	470	466	462	− 5
	457	451	446	440	434	− 7
	427	420	413	405	397	− 8
	139389	381	372	362	353	−10
	343	333	322	311	300	−11
	289	277	265	252	239	−13
	226	212	198	184	169	−15
	154	139	123	107	091	−17
	139074	057	040	022	004	−19
	138985	967	947	928	908	−20
	888	868	847	826	804	−22
	782	759	737	714	691	−24
	667	642	618	594	568	−25
	138543	517	491	464	437	−27
	410	382	355	326	298	−29
	269	239	209	179	149	−31
	118	087	055	023*	991	−33
	137958	925	892	858	824	−35

Table III.

		η.								
ξ.		0.	1.	2.	3.	4.	5.	6.	7.	8.	9.
0·00	1·0	00000	00998	01992	02982	03968	04951	05930	06904	07875	08843
0·01		09807	10767	11724	12677	13626	14572	15515	16454	17390	18322
0·02		19252	20178	21100	22020	22936	23849	24759	25666	26569	27470
0·03		28368	29262	30154	31043	31929	32811	33692	34569	35443	36315
0·04	1·0	37183	38049	38913	39773	40631	41486	42339	43189	44036	44881

The value of the function $G(\xi)$ may be found in the tables with argument ξ (Table I.) or ξ' (Table II.); the root of the equation (2) may be found in Table III. with argument $\zeta = m \cdot G(\xi)$. Having determined the value η_i, one may compute $\sqrt{p}$ from the formula

$$\sqrt{p} = \frac{n_i \eta_i}{k \tau_i}.$$

The scheme proposed permits us to avoid successive approximations, and may be applied to any conic section and any interval of time between the observations. The tables, however, may be used even with the small intervals $-\tau_i$. In the case of small τ_i the exactness of the value ξ_i, determined by the formula (1), is comparatively small, but this affects but little the value of the function $G(\xi_i)$.

A larger table of this kind is to be found in vol. ii. of the *Publ. of the Tashkent Astr. Obs.*, 1929 (Subbotin, "Formules et tables"), Table XXIII. (In this log η^2 is given with argument ζ.)

Voronezh University:
1929 *December.*

Visual Observations of Dark Nebulæ. By W. S. Franks.

Sir John Herschel, in *Cape Obs.*, gave a list of "starless fields," where blank gaps were found among crowded fields of stars, chiefly connected with the galaxy; but he apparently did not perceive their real character. This was first recognised by Secchi as being probably due to obscure nebulosity intercepting the light of stars beyond; and he mentioned a particular dark nebula in Sagittarius, which he discovered in 1877, as an illustration of this. Seven years later this same object was rediscovered by Barnard, who called it a "black hole." But as a result of more extended experience he came round to the view that these curious vacancies were not "holes," but really *dark nebulæ*; and in 1919 he published a list of 182 such dark markings derived from his Milky Way photographs. Several of these were also discovered visually by him; as also one by Espin (*M.N.*, **58**, 334, 1898), whilst sweeping for red stars with a 17¼-inch reflector. Then Max Wolf found a "lacuna" or dark ring surrounding a nebula in Cygnus (I.C. 5146) which he photographed (*M.N.*, **64**, 838, 1904); and this was connected with a dark lane of considerable length running westward from it. This latter is also Barnard 168, but is not included in his later charts. Barnard examined with the 40-inch Yerkes refractor some of the smaller dark spots on his photographs, and in every case found there was an overspreading feeble nebulosity, which was conclusive evidence of their nebulous character. Among the dark markings noted by him are many "dark lanes," often of considerable length, between rich areas of stars, some of which are pretty certainly due to obscuring nebulosity. Since the publication of his beautiful *Photographic Atlas of the Milky Way*, it

occurred to me to examine visually some of these dark spots and vacancies, which might be within the range of a 6-inch refractor. As a result of some preliminary observations, it was suggested by Father Hagen that I might profitably undertake a more or less systematic survey of these objects; and he very kindly offered me the loan of a duplicate set of the key charts and catalogue for the purpose of ready identification, which offer I gratefully accepted.

The following observations, therefore, were made with the 6-inch Cooke refractor at Brockhurst, usually with a fine achromatic Kellner eyepiece of 1-inch focus and field of 36′. Trial was also made of a micro-objective of similar power, used as a single lens, with its convex side towards the eye, to diminish spherical aberration. This was effective enough as regards flatness of field, achromatism, and crisp definition, but its small field was a great drawback. There was, however, a slight gain in light over the two-lens eyepiece, which sometimes proved useful on very faint objects. As it was not to be expected that I could see certain of the difficult objects visible in the 16-inch refractor at Rome, either on account of their small size or feeble density, a good deal of pruning was done in preparing the working list. Out of Barnard's 349 dark objects traceable on the photographs there are some 120 or so which are only mere spots of small diameter; these are generally too small to be detected in the telescope. Moreover, though on the photographs they are sharply defined, yet to the eye they are very diffuse and sometimes unrecognisable. And it must also be borne in mind that as comparatively few out of the immense number of stars shown by the photographs can be seen in the telescope, the apparent contrast is thereby greatly diminished, and so the ultimate effect on the eye is very different in the magnified image. With regard to the southern objects, the difference of 10° from the lower latitude of Rome made it necessary to curtail these observations to a limit of 28° South declination. As the richest part of the galaxy is situated south of the equator, more especially in the constellations Scorpio, Ophiuchus, and Sagittarius, some two-thirds of Barnard's objects are found within the limits of 16^h to 19^h in R.A.; but from 6^h to 16^h there is a blank. This crowding together in a limited space made the work of observing more difficult, and many objects had to be missed in consequence.

All the objects in the following list were observed on two nights, commencing in 1929 August and ending in December. They are given in R.A. order, beginning at 16^h and ending at 6^h. It has not been thought necessary to give the approximate co-ordinates of these dark nebulosities, since they can be found in Part II. of Barnard's work; nor those of the guiding stars, reduced to epoch 1925, which appear in Father Hagen's "Notes" in Part IV. of *Miscellanea Astronomica* (Spec. Vat.). The Barnard number is therefore deemed sufficient for identification; but generally the description of the object is supplemented by its bearing from an adjacent catalogue star or cluster. It seems scarcely necessary to add that these observations were made only on very clear nights, and in the total absence of moonlight, when the Milky Way was conspicuous.

Observations.

Barnard.

40. A dusky cloud, about $\frac{1}{4}°$ diameter, of a peculiar dingy tint, margin diffuse, 8·5 mag. star (BD − 18° 4247) on *np* edge. About $\frac{3}{4}°$ *nf v* Scorpii.

41. Very large dense cloud *np* ψ Ophiuchi; too large for field of 36′; like an opaque veil, not a star to be seen. Identical with one of Herschel's starless fields.

42. An obscure spot in a dark gap about $\frac{1}{2}°$ *sp* ρ Ophiuchi. There is a great blank space *f* that star, which is also a Herschel region.

44. A very long, broad, dark lane, running eastward from 22 Scorpii. Not very easy to follow, from its great size; much sweeping necessary.

45. Two parallel dark lanes, about 2° long and $\frac{3}{4}°$ apart, running in a *sp* to *nf* direction. An 8 mag. star (BD − 21° 4403) lies between them, towards the *f* end. Fairly definite, and not difficult.

78. A large vacant region in Ophiuchus, which is visible to the naked eye, CD − 26° 12152, 6·2 mag. is near the middle. One of Herschel's blank regions.

86. A feeble dingy cloud in oval ring of stars, its *n* edge touching the cluster N.G.C. 6520; the latter curiously placed at the *s* end of a chain of stars running *np* and ending in a brighter one. [This is Secchi's dark nebula.]

88. This is one of several dark gaps between the masses of bright nebulosity in N.G.C. 6523 (M8). These were noticed independently last year, when using a 24-inch reflector, in which M8 was a striking and beautiful object.

92. A dusky oval spot in a clear gap *np* the cluster N.G.C. 6603 (M24 Scuti). Rather difficult, except for the contrast with surrounding clear space.

95. A peculiar dark lane, about 1° long, of a nondescript dingy hue. It is about $\frac{1}{2}°$ *nf* BD − 12° 5024, 5·5 mag.

97. An oblong dense patch, larger than the field of view. Pretty dark, though indefinite. It lies about $1\frac{1}{2}°$ *sf* ζ Scuti.

100. A dark streak, running diagonally from *np* to *sf*. About 2° *f* ζ Scuti.

101. A similar streak, running from *sp* to *nf*, and forming an elbow with B100. There are some small stars at the juncture.

318. A long very narrow lane, extending over 1° *p* and *f*, between the cluster N.G.C. 6705 (M11) and Barnard 112. Distinctly seen, though so narrow.

112. A diffuse dark spot, $\frac{1}{4}°$ *s* of M11. Pretty readily seen.

320. Irregular patchiness in field, around a 7·5 mag. star (BD − 6° 4941).

323. A pretty dark patch, about $\frac{1}{4}°$ diameter, surrounded by many stars. About $1\frac{1}{2}°$ *f* BD − 3° 4392, 6·5 mag.

324. Very peculiar, crooked, dark lane, in shape resembling the letter J, with a star of 7·4 mag. in the bend (BD − 3° 4439).

Barnard.

135. A dusky patch, about $\frac{1}{4}°$ diameter. About 1° *n* of λ Aquilæ.
136. Another dark spot, smaller but denser, closely *f* the last.
141. Roundish spot in rather bare region ; BD + 1° 3967, 8·9 mag. in field.
330. Dark patch, about $\frac{1}{2}°$ diameter; BD + 7° 4028, 7·8 mag. on *nf* edge.
333. Dark area of irregular markings, including 7·5 mag. *nf* centre (BD + 10° 3913).
142. Large and irregular, surrounded by rich area. About $1\frac{3}{4}°$ *p* γ Aquilæ.
143. Large curved dark marking, somewhat the shape of a horse-shoe, in very rich region of stars. About $\frac{1}{2}°$ *nf* the last object.
339. Large, irregularly dark region, with 7·0 mag. star in centre (BD + 8° 4200), and another 7·0 mag. on *sp* edge (BD + 8° 4190).
340. Curious angular marking, directly *s* of and nearly touching π Aquilæ.
341. Narrow dark line along parallel, directly *s* of and pretty close to BD + 33° 3615, 8·4 mag. Distinctly seen.
144. Long vacant region *nf* η Cygni ; very few stars in it, but densely crowded all around.
145. Curious triangular dusky marking, extended about $\frac{3}{4}°$ *p* and *f*, with 7·0 mag. star (BD + 37° 3735) on its *n* base. Quite distinct.
343. Dusky oval cloud, about 15′ diameter. Closely *sf* BD + 39° 4075, 6·8 mag.
345. Dusky curve, convex to *f*. About $\frac{1}{2}°$ *nf* BD + 45° 3139, 6·6 mag.
346. Another curved marking, convex to *s*, small star to the north. About $\frac{1}{3}°$ *sp* BD + 43° 3609, 8·1 mag.
348. Peculiar large curved dusky lane, something like a shepherd's crook, with bright star in the bend (BD + 41° 3799, 6·7 mag.). Well seen.
349. Small curved dusky marking, rather faint. About $\frac{1}{2}°$ *p* 56 Cygni.
150. Long curved dark marking ; BD + 59° 2291, 8·0 mag. at *nf* end.
351. Dusky lane, *sp* to *nf*. About $\frac{1}{2}°$ *p* BD + 46° 3111, 6·0 mag.
352. Large dense patch, extended *p* and *f*, small stars at *f* end. About 2° *f* 55 Cygni.
353. Small but distinct dusky spot, $\frac{1}{2}°$ *s* of last.
354. A crooked irregular dusky lane, about 1° long, *sp* to *nf*, with stars on either side. About 1° *n* of BD + 56° 2523, 6·5 mag.
356. Pretty large but irregular dusky patch, well marked by being encircled with small stars. About $\frac{3}{4}°$ *s* of 59 Cygni.
357. Dense dark space, nearly filling field of 36′ diameter. BD + 55° 2495, 7·9 mag., is on the *np* edge.

Barnard.

358. Very diffuse dusky patch, extended *p* and *f*. About 40′ *s* of ξ Cygni, a little *f*.

360. Irregular dark marking, about ½° *sp* to *nf*. About 6ᵐ *f* BD + 56° 2524, 5·8 mag.

361. A round dusky patch, fairly well seen. About ¾° *sp* BD + 47° 3348, 6·3 mag.

362. Dark oval marking, extended *sp* and *nf*; 9 mag. star at *nf* end. It is situate about ½° *sf* BD + 50° 3317, 8·2 mag.

363. Irregular broad dark marking, right across field, from *sp* to *nf*. A bright star, BD + 48° 3390, 5·5 mag., is near its *sf* edge.

365. Dark crooked marking, roughly **S** shape, about 20′ *n* to *s*. It is about 1° *s* of and 4ᵐ *p* BD + 56° 2617, 6·2 mag.

160. A large blank area, about ½°. It lies about ½° *sf* B365.

366. Rather small, round, dusky spot. About ¼° *s* of BD + 59° 2409, 6·9 mag.

164. Curious angular dark marking, like inverted **V**, ⅓° *n* to *s*. 1° *f* π Cygni.

168. Narrow dark lane, extending *np* from Max Wolf's nebula (I.C. 5146) through several fields. [Not on charts.]

169–170. "Island" of small stars, bordered by dark ring. B170 is the densest part of it, on *n*. About 1° *np* 14 Cephei, 6·4 mag.

174. Narrow crooked marking, about ⅓° *n* to *s*. A bright double star (Σ2872) is in this field, a little to the *n*. About ½° *sp* λ Cephei.

201. A small dusky cloud, isolated in clear space and fairly easy to see. It is situated about ¾° *p* the first of the great clusters in Perseus (N.G.C. 869).

202. Dark lane, about ½° long, running diagonally from *np* to *sf* (½° *s* of B203).

203. Narrow dark strip, with 9·0 mag. star on *n* edge. About 10′ *s* of BD + 30° 540, 8·8 mag.

204. Irregular dark patch, *np* to *sf*. About ¼° *sp* BD + 29° 566, 6·8 mag.

1. Blank area, about size of field; 1° *p* BD + 30° 558, 7·1 mag.

2. Dark streak *np* to *sf*, with two stars involved; BD + 31° 616, 6·5 mag. at *np* end, and BD + 31° 619, 7·0 mag. at *sf* end.

3. Small dark gap, with faint nebulosity on *p* side. About 1° *p* *o* Persei.

4. Large blank area, about 1° *s* of *o* Persei.

5. Broad dark streak, reaching from η Persei about 1° towards *sp*.

7. Large dark area, containing B10 as densest spot; *p* and *n* of φ Tauri.

10. The densest part of B7. Small, well-defined dark spot. About 1° *np* φ Tauri.

18. Irregular patchy field, with star involved at *sf* end (BD + 23° 705, 8·0 mag.).

20. Small dingy cloud, near group of small stars. About 4ᵐ *f* Σ553, a little *s*. [This is the object independently dis-

Barnard.

covered by Espin. Not on charts.] The inversion of numerical order in this case is due to precession.

219. Barren region, very large and indefinite; few stars. About 1° *n* of BD + 28° 666, 5·7 mag.

19. A large dark area, part of a long sinuous lane; nearly 1° *sp* Σ572 (6·5, 6·5).

22. Another dark area in same lane, about ½° *f* the last.

221. A rather bare region, larger than field of 36′. About 1° *sp* BD + 31° 815, 9·0 mag.

224. A dark patch, about half diameter of field. About ¾° *sf* BD + 10° 760, 7·0 mag.

225. A dark extension of about ¾° from B30. Directly *s* of BD + 12° 801, 6·9 mag.

30. A large dark area, with few stars; about ½° *f* BD + 12° 801.

31. A darker patch of B30, a little to the *nf*.

32. An extension from the *s* end of B31, towards the *f* side.

226. A dark spot, about ¼° diameter; ½° *s* of the cluster N.G.C. 1960 (M36).

34. Another dark patch, rather indefinite; ½° *sf* BD + 32° 1064, 7·0 mag.

35. Oval dark streak, tapering to *f* end, with a 9·5 mag. star on *n* edge. It is about 1° *f* BD + 9° 925, 7·5 mag.

36. A dark lane about 2° long, and rather narrow, running from *sp* to *nf*. It is about 1° *n* of 52 Orionis, at its *sp* end.

227. A dark round spot, about 1° *f* 64 Orionis, a little to the *s*.

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On the Significance of Baxendell's Nebulosity.

By J. G. Hagen, S.J., Assoc.R.A.S.

Baxendell discovered his nebula (*N.G.C.* 7088) half a century ago. Since then very little attention has been paid to it, although it is closely attached to the bright globular cluster *Messier* 2. Baxendell's letter to W. H. M. Christie is printed in *M.N.*, **41**, 48 (1881). It says: "I have seen it [the nebula] on several nights and have no doubt of its existence." The instrument was probably the 5-inch refractor mostly used by Baxendell in his variable-star work.

The first astronomer who afterwards looked at the nebula was Dr. Dreyer. In a note to his *N.G.C.* (p. 225) he says: "I have seen it without difficulty in the Armagh 10-inch refractor." The nebula is also mentioned by Bigourdan in his "Observations," where "quelques traces de nebulosité" were suspected.

1. The nebula is described by Baxendell in these words: "It seems to be similar in character to the large Nebula near the Pleiades, but is slightly less bright." Dreyer calls it "a very large and very diffused nebulosity."

It belongs in fact to the class of Herschel's extensive milky nebulosities and of Barnard's "Dark Objects," though not found in either of their catalogues. Nor is it mentioned in the *Publications of the Lick Obs.*, vols. xi. (1913) and xiii. (1918), nor in the *Cordoba Photographs*, vol. xix. (1897).

In the course of the Vatican review of the *N.G.C.*, Baxendell's nebulosity was observed in the years 1915 and 1917 as "obscure" and of density III–IV (on a scale of five). The observation was independently confirmed by Dr. Fr. Becker at Potsdam. In the Vatican *Zone Catalogue* * it is described as *np* attached to the globular cluster *Messier* 2, elongated in a *pf* direction and extending over 0·8 by 1·0 degrees. It is not found in the Vatican *General Catalogue*, because all the obscure nebulæ of Dreyer's *N.G.C.*, seventeen in number, are relegated to a special Table VI., p. xxiv. of the Introduction.

The co-ordinates of cluster *Messier* 2 for 1930 are $21^h\ 30^m$ and $-\ 1°\ 8'$. It comes into convenient position in the months of September, October, and November. This year (1929) the writer called attention to it in the *Astr. Nachr.*† Several observers reported by letter that they saw the nebula quite clearly, at Brockhurst, Potsdam, and Stonyhurst, with refractors from six to fifteen inches aperture. Special credit is due to Father O'Connor's observations because, after finding Baxendell's nebula "without difficulty" in the Stonyhurst 15-inch refractor, Father O'Connor discovered another extensive nebula about one degree further south, and even a third one, half a degree in a *p* direction from the second. The clear dark sky between the three nebulæ and their difference in density gave him the guarantee that there could be no illusion. The second and third nebulæ are genuine Herschel fields. Their densities, as estimated at Rome, are IV–V for the second, and V for the third nebulosity. The observer will readily find all three, if his field of view is not too small.

2. Although Baxendell's nebula is the faintest of the three near the cluster *Messier* 2, its peculiar significance is in the line of photography. As it is closely attached to a globular cluster, photographs of that region have been taken before, though none was ever published.

Dr. Shapley, kindly responding to a letter, sent us prints from one of the best plates of *Messier* 2 taken with the 60-inch reflector at Mount Wilson, with 55 minutes exposure, by Professor Duncan in 1919. The stars of the cluster came out with wonderful sharpness, but of nebulosity there is no trace.

An exposure of 12 hours was made by M. de Kerolyr in his mountain observatory at Digne (B.-A.) with an Opto-Lumière plate. The print which M. de Kerolyr sent us shows no trace of nebulosity.

Attention had been called to Baxendell's nebula by Dr. Fr. Becker in the *Astr. Nachr.*‡ in 1925. It did not remain unheeded. Director Schorr of the Hamburg Observatory reports in the *V.J.S. der Astr. Ges.*§ that Dr. Baade made exposures with the large reflecting telescope

* *Spec. Vat.*, **10**, 324 (1922). The diameters should be in heavy type.
† *Beobachtungs Zirkulare*, 1929, Nos. 30, 34.
‡ Band 224 (1925), 115/6. § Band 64 (1929), 198.

without obtaining any impression of the nebula. The exposures were made first with ordinary plates, then with red-sensitive plates, and finally with panchromatic plates. The light was made to pass through a yellowish filter, except in the first case. The exposure times varied from 90 minutes to 4 hours. Though the result was negative, Dr. Baade intends to continue his experiments with plates of still higher sensitivity in the long-wave spectral region.

The short notice of the writer in the two circulars of the *Astr. Nachr.* induced Dr. Shapley to have some special plates taken at Harvard around the region of Baxendell's nebula. He kindly sent us a print from a plate of 90 minutes exposure, made with the 16-inch Metcalf telescope, using a yellow-sensitive plate and a yellow filter. The result was likewise negative. So also was the result of two plates taken at Helwan with the 30-inch reflector and exposures of 60 and 70 minutes.

3. It may be questioned whether we are on the right way to solve the problem. It seems quite possible that the secret lies in the developer. All our astrophotographic methods operate with light coming exclusively from stars, either directly or indirectly. There are, however, certain photoluminescences whose chemical process is still involved in deep mystery. The light of extensive nebulosities may well have impressed many plates, but may just as well have remained latent because the specific developer was not applied.

Instead of spending hours in exposing plates, would it not be advisable to expose a dozen plates on one night, and to develop them all in different ways? Chemical science will have to come to the astronomer's aid.

The trial could even commence in daytime with terrestrial clouds when they cover the whole sky and are not illuminated from the outside. Their greyish tint presents a striking resemblance to Herschel's nebulosities. Existing photographs seem to show that terrestrial clouds have never been photographed in *their own* light. A useful study in this respect can be made on a photograph of a thunderstorm made at Toronto by the U.S. Weather Bureau, and reproduced in the new edition of the *Encyclopedia Britannica*.* The dark patches of that picture might have shown some light if developed in a different manner.

At the Heidelberg meeting of the *Astr. Ges.* it was mentioned † that "the crucial test" of the existence of Herschel's nebulosities will be a photographic trial by means of visual-colour filters. Does not Baxendell's nebulosity seem to reverse the case? Its existence is so well assured by experienced observers that it may with some right present itself as a crucial test for what our present photographic skill is able to accomplish. Indeed, Baxendell's nebula seems now to invalidate the objections that are raised, from the photographic point of view, against the reality of Herschel's milky nebulosities.

Vatican Observatory:
1929.

* Volume 14, facing page 114.
† *V.J.S. der Astr. Ges.*, **63** (1929), 322.

Reduction of Occultations observed in 1928. By B. M. Peek.

During 1928 I observed twenty-eight occultations of stars by the moon, four of them with the 8½-inch reflector at Mr. C. S. Saxton's observatory at Frome in Somerset, and the remainder with the 150-mm. Zeiss refractor at my observatory at Herne. The co-ordinates of both observatories are given in *N.A.* for 1929.

I have since reduced the observations, using the method given by Innes,* and the operation has been performed in duplicate in order to make the detection of error tolerably certain. In accordance with the recommendation of Professor E. W. Brown, the moon's place and parallax have been taken from the ephemeris for a time 0·212 minutes later than that actually observed, which is equivalent to applying an average correction of about 7 seconds of arc to its tabular longitude.

The star places have been taken whenever possible from *N.A.*, which, with the exception of ϵ Geminorum, gives the positions as taken direct from W.Z.C., which is accordingly quoted as the authority in Table I. The remaining positions have been very kindly collected from the various catalogues at no small expense of labour, weighted, and forwarded to me by Dr. L. J. Comrie and Mr. T. Whitwell, and are the best available for the stars in question ; these are referred to as C.W., a numerical suffix up to 5 denoting the weight assigned to each position.

All the phenomena recorded are disappearances, all but two of which took place at the dark limb. My usual method of recording by means of a stop-watch registering tenths of a second was employed, and normally the times have been corrected by $-0^{s}\cdot5$ to allow for lag in starting the watch ; as, however, it is usually possible to estimate departures from the normal in starting, this correction has often been modified in accordance with the impression created at the time of observation.

For convenience of printing, the results are shown in two tables, Table I. giving the particulars of the observation, star place, etc. ; Table II. the results of the reduction and remarks.

It seems worth noting that a rough inspection of the values of $\sigma' - \sigma$ does not lead one to conclude that the average residuals for stars below 8th magnitude are any greater than for the brighter stars. It therefore seems well worth while to include these fainter stars in any observing programme. The star B.D. + 16° 2106, mag. 9·3, whose disappearance was observed with the 6-inch refractor on May 25, was about as faint as could have been observed under the existing conditions, the moon's altitude being only about 15° and age 6½ days. Under favourable circumstances far fainter stars could be observed if reliable places for them were available.

The disappearance of W.Z.C. 695 on 1928 April 28 was remarkable. I was using the 8½-inch reflector at Frome, and Mr. Saxton a 3-inch refractor about 25 yards south of the 8½ inch. The disappearance for me was very gradual, lasting for about 0·75 seconds, while Mr. Saxton recorded no gradual fading, but a preliminary jump more than a second before

* Union Observatory Circular No. 56, 1922, Sept. 12.

final disappearance. Critical examination of the star at a later date revealed no trace of duplicity, and when I again observed its disappearance on 1929 March 22 it was absolutely instantaneous. Evidently we have here an example of the parallactic displacement relative to the star of some irregularity of the moon's limb due to an extremely short base line. The curious thing about the phenomenon is that the occultation was by no means a grazing one, $\chi - \rho$ being only $14\frac{1}{2}^\circ$, so that a very jagged or precipitous lunar crag must have been involved.

TABLE I.

No.	Date. 1928.	G.M.T. h m s	Star.	Mag.	R.A. to Date. h m s	Dec. to Date. ° ′ ″	Authority.
1	Jan. 3	16 39 41·1	33 B Tauri	6·3	3 35 21·09	+16 18 13·7	W.Z.C.
2	27	17 51 54·0	B.D.−1° 94	8·3	0 42 2·44	− 1 34 53·0	C.W.$_3$
3	27	20 19 37·3	W.Z.C. 45	6·8	0 46 12·43	− 0 37 7·8	C.W.$_4$
4	Feb. 25	18 17 6·9	B.D.+9° 296	8·2	2 13 0·72	+ 9 27 4·5	C.W.$_3$
5	25	19 6 31·2	B.D.+9° 301	8·5	2 14 39·22	+ 9 30 55·8	C.W.$_2$
6	25	20 21 43·0	B.D.+9° 306	8·3	2 17 0·74	+ 9 40 31·8	C.W.$_4$
7	25	21 57 50·0	W.Z.C. 138	7·3	2 20 17·71	+ 9 56 44·0	C.W.$_3$
8	26	20 52 4·6	B.D.+15° 459	8·9	3 12 39·81	+15 23 45·0	C.W.$_3$
9	27	18 12 49·5	43 Tauri	5·5	4 4 57·44	+19 25 12·2	W.Z.C.
10	27	20 9 50·0	B.D.+19° 675	7·8	4 8 37·05	+19 21 18·6	C.W.$_2$
11	Mar. 26	19 41 59·6	B.D.+21° 723	8·4	4 50 29·93	+22 1 52·0	C.W.$_2$
12	26	20 28 34·3	B.D.+22° 776	7·5	4 51 45·95	+22 27 51·0	C.W.$_3$
13	28	23 57 30·6	B.D.+25° 1571	7·0	7 1 47·15	+24 58 19·1	C.W.$_2$
14	Apr. 2	20 38 4·8	W.Z.C. 747	6·8	11 22 35·37	+ 9 3 16·5	C.W.$_4$
15	24	19 58 21·4	B.D.+25° 1362	8·5	6 34 59·10	+25 8 16·4	C.W.$_2$
16	24	20 18 27·2	B.D.+25° 1372	8·5	6 35 34·91	+25 13 54·3	C.W.$_2$
17	24	20 26 14·1	B.D.+25° 1378	8·8	6 36 0·55	+25 8 42·6	C.W.$_2$
18	24	22 2 53·4	ε Geminorum	3·2	6 39 29·39	+25 12 18·7	N.A.
19	24	22 38 42·1	B.D.+24° 1386	7·8	6 40 58·01	+24 44 55·6	C.W.$_3$
20	28	22 4 47·5	W.Z.C. 695	7·1	10 24 58·38	+14 42 43·1	C.W.$_2$
21	29	0 8 5·0	46 Leonis	5·8	10 28 21·92	+14 30 26·3	W.Z.C.
22	May 2	23 53 25·8	*l* Virginis	4·8	13 28 14·32	− 5 53 12·1	W.Z.C.
23	25	22 58 34·3	B.D.+16° 2104	8·5	10 12 7·02	+15 45 22·9	C.W.$_1$
24	25	23 33 51·6	B.D.+16° 2106	9·3	10 13 22·11	+15 52 45·3	C.W.$_2$
25	July 29	23 0 23·6	68 G Sagittarii	6·2	18 23 16·84	−26 40 47·2	W.Z.C.
26	29	23 41 24·9	86 B Sagittarii	6·5	18 24 30·52	−26 37 45·3	C.W.$_4$
27	Aug. 10	2 49 40·8	224 B Tauri	6·1	4 18 9·06	+20 39 14·6	W.Z.C.
28	10	3 26 8·9	227 B Tauri	5·9	4 19 18·28	+20 49 1·3	W.Z.C.

[TABLE

TABLE II.

No.	$\sigma'-\sigma$.	$\chi-\rho$.	Sin $(\chi-\rho)$.	Cos $(\chi-\rho)$.	Remarks.
	″	° ′			
1	−0·25	6 32·7	+·1140	·9935	Frome.
2	+1·67	66 16·0	+·9154	·4025	
3	−0·94	348 7·3	−·2058	·9786	
4	−0·36	348 20·5	−·2021	·9794	
5	−2·60	15 4·6	+·2601	·9656	Small preliminary dip, just before total extinction.
6	+0·61	38 14·0	+·6189	·7855	
7	−0·96	52 59·5	+·7985	·6019	
8	−0·47	329 37·6	−·5056	·8627	Record altered + 1 minute.
9	−1·20	330 54·4	−·4862	·8738	
10	+2·65	48 24·3	+·7479	·6639	
11	+0·75	43 43·4	+·6912	·7227	
12	−0·79	313 37·1	−·7240	·6898	Disappearance sensibly gradual.
13	−0·74	302 55·6	−·8394	·5436	
14	−1·31	350 17·6	−·1686	·9857	A flicker immediately preceded disappearance.
15	−1·60	355 43·2	−·0746	·9972	
16	+0·05	331 17·5	−·4804	·8771	
17	+0·73	349 27·5	−·1830	·9831	
18	−0·72	311 25·0	−·7499	·6615	
19	+0·34	53 28·2	+·8035	·5952	Star glided out.
20	−0·15	14 29·2	+·2501	·9682	Frome. Remarkable disappearance; see text. Time uncertain ± 1 second.
21	−1·38	318 46·4	−·6590	·7521	Frome.
22	−0·73	349 44·3	−·1781	·9840	Frome.
23	−1·11	13 48·8	+·2388	·9711	
24	−1·62	314 17·5	−·7158	·6983	
25	−0·73	25 57·9	+·4378	·8991	Image dimmed by cloud about the time of disappearance, but little doubt as to accuracy of observation.
26	−4·03	15 12·2	+·2622	·9650	Star place very carefully examined by C. and W.; but $\sigma'-\sigma$ still large. Query, failure of stop-watch.
27	−0·92	343 4·4	−·2911	·9567	Bright limb. No allowance for personal equation.
28	−0·81	332 36·7	−·4600	·8879	Bright limb. No allowance for personal equation.

Occultations of Stars by the Moon, observed at Birmingham University during the Years 1928 *and* 1929. By James Young.

No.	Date.	G.M.T. h m s	Star.	Mag.	Phase.
1	1928 Jan. 27	20 17 49·5	W.Z.C. 45	6·8	D
2	Feb. 27	18 10 58·9	43 Tauri	5·5	D
3	Apr. 24	22 0 20·2	ε Geminorum	3·2	D
4	Oct. 25	19 57 1·3	W.Z.C. 1578	7·3	D
5	25	21 6 46·7	W.Z.C. 1580	6·8	D
6	1929 Jan. 16	18 32 9·3	B.D. −3° 38	7·0	D
7	16	20 58 6·0	54 B Ceti	6·3	D
8	Feb. 13	18 46 5·6	W.Z.C. 51	7·3	D
9	17	20 4 54·7	B.D. +22° 712	7·0	D
10	Mar. 16	22 26 6·6	κ Tauri	4·1	D
11	16	22 32 36·4	67 Tauri	5·4	D
12	22	19 19 23·0	42 Leonis	6·1	D
13	Apr. 14	21 7 18·7	W.Z.C. 421	7·4	D
14	15	19 21 50·8	B.D. +26° 1481	7·0	D
15	16	21 50 4·3	B.D. +24° 1903	7·0	D
16	16	22 27 27·4	λ Cancri	5·9	D
17	18	18 19 6·5	η Leonis	3·6	D
18	18	19 27 41·5	η Leonis	3·6	R
19	21	21 9 1·0	Lalande 23625	7·0	D
20	21	23 17 43·9	γ Virginis	2·9	D
21	21	23 17 58·0			
22	May 12	22 2 13·4	39 Geminorum	6·2	D
23	Oct. 24	23 3 48·2	4 Cancri	6·2	R
24	Nov. 14	20 44 7·9	W.Z.C. 96	6·9	D
25	14	21 21 25·7	W.Z.C. 97	6·7	D
26	Dec. 16	21 52 40·1	B.D. +26° 1082	7·0	R
27	18	20 14 30·2	λ Cancri	5·9	D
28	18	21 5 31·7	λ Cancri	5·9	R
29	19	21 45 43·7	W.Z.C. 637	6·6	R

Instrument used :

7-inch Cooke refractor.

Position of observatory :

Latitude	N. 52° 26′ 58″·3
Longitude	W. 0^h 7^m $42^s·49$
Altitude	460 ft.

Notes.

The observations have been corrected for personal error and should in most cases give the time of disappearance or reappearance to within $0^s·2$.

No. 24. This star is a double. Observation is for second disappearance.

No. 27. Disappearance at bright limb. Probable error about 1^s.

No. 29. Probable error about 1^s.

Occultations of Stars by the Moon, observed at the Nizamiah Observatory, Hyderabad, during the period 1928 November–1929 June.

(*Communicated by the Director.*)

No.	Date.	Star.	Mag.	Phase.	Greenwich Mean Time (new). h m s	Observer.
1	1928 Nov. 19	B.D. −22°·5667	9·3	D	12 53 36·1	B
2	19	A.G. Algiers 9145	8·8	D	13 11 23·7	B
3	19	B.D. −21°·6000	9·6	D	14 33 51·3	B
4	19	B.D. −21°·5998	9·2	D	14 34 6·3	B
5	19	B.D. −21°·6004	9·5	D	15 8 2·1	B
6	19	33 Capricorni	5·3	D	15 38 58·8	B
7	Dec. 18	A.G. Wash. 8453	8·9	D	13 27 56·5	B
8	18	A.G. Wash. 8460	9·0	D	13 34 46·0	B
9	19	A.G. Wien 8357	9·1	D	14 29 6·2	M
10	19	B.D. −8°·6151	10·0	D	15 51 47·2	M
11	19	A.G. Wien 8360	9·0	D	16 1 9·8	M
12	20	B.D. −3°·46	9·2	D	13 5 52·1	B
13	20	54 B Ceti	6·3	D	14 6 29·4	B
14	25	103 Tauri	5·5	D	14 30 6·0	M
15	1929 Jan. 16	B.D. −4°·3	9·5	D	13 14 58·0	B
16	16	A.G. Strassburg 24	9·2	D	13 19 28·5	B
17	17	A.G. Albany 240	7·9	D	13 51 48·8	B
18	17	A.G. Albany 248	8·9	D	14 35 6·7	B
19	17	B.D. +1°·182	9·3	D	14 41 21·7	B
20	17	A.G. Albany 253	8·0	D	14 54 7·3	B
21	17	A.G. Albany 258	8·6	D	15 18 18·5	B
22	20	Berlin A 994	9·1	D	17 40 19·2	M
23	20	14 Tauri	6·2	D	17 50 15·3	M
24	20	13 Tauri	5·6	R	18 17 31·6	M
25	22	139 Tauri	4·7	D	21 37 15·8	M
26	30	72 Virginis	6·1	D	21 29 38·3	M
27	Feb. 17	*ν* Tauri	4·2	D	17 52 35·9	M
28	17	*ν* Tauri	4·2	R	18 55 34·7	M
29	Mar. 16	A.G. Berlin 1311	8·6	D	13 58 43·1	M
30	16	A.G. Berlin 1312	9·0	D	14 8 13·0	M
31	16	B.D. +21°·589	9·5	D	14 28 58·1	M
32	16	A Tauri	4·5	D	14 39 17·2	M
33	16	39 Tauri	6·1	D	14 40 30·2	M
34	16	A.G. Berlin 1320	8·2	D	14 48 19·7	M
35	16	A Tauri	4·5	R	15 8 8·4	M

No.	Date.	Star.	Mag.	Phase.	Greenwich Mean Time (new).	Observer.
					h m s	
36	1929 Mar. 16	A.G. Berlin 1329	8·0	D	15 12 37·4	M
37	22	42 Leonis	6·1	D	21 23 5·6	M
38	April 15	A.G. Cambridge 3603	9·2	D	13 46 26·0	M
39	15	A.G. Cambridge 3626	8·3	D	14 46 33·3	M
40	15	A.G. Cambridge 3657	7·8	D	15 53 55·6	M
41	16	ω Cancri	6·1	D	14 22 51·5	M
42	18	η Leonis	3·6	D	20 43 0·8	M
43	24	8 Librae	5·4	D	17 6 20·9	M
44	24	α Librae	2·7	D	17 16 51·5	M
45	24	8 Librae	5·4	R	18 33 12·4	M
46	May 4	336 B Aquarii	6·3	R	23 3 12·7	M
47	13	A.G. Cambridge 4121	9·4	D	14 35 15·5	B
48	15	57 B Leonis	6·5	D	15 25 22·9	M
49	June 12	B.D. +16°·2114	9·2	D	15 14 34·4	M

Instrument used :

15-inch Grubb refractor, except for No. 42, which was observed with a 3-inch portable telescope.

Observers :

B = T. P. Bhaskaran.
M = M. K. Bappu.

Notes.

24. Slightly late.
26. Observation doubtful owing to clouds.
42, 47. Sky hazy.
28, 41, 43, 44, 45, 48. Seen through clouds.

Nizamiah Observatory,
Hyderabad (Deccan),
India.